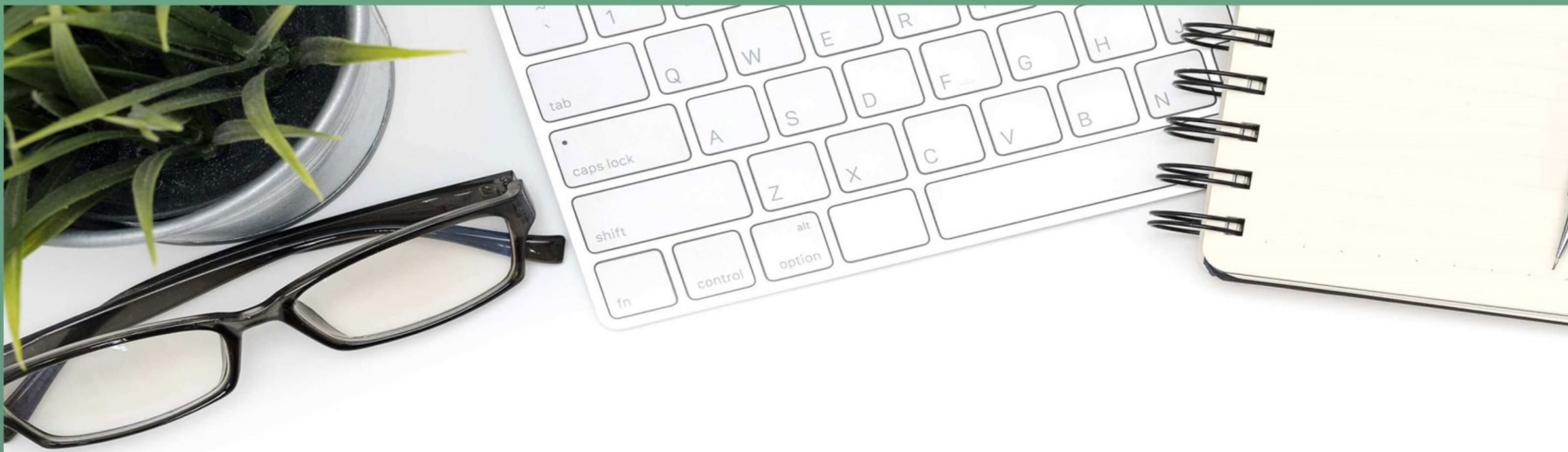

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Important Questions

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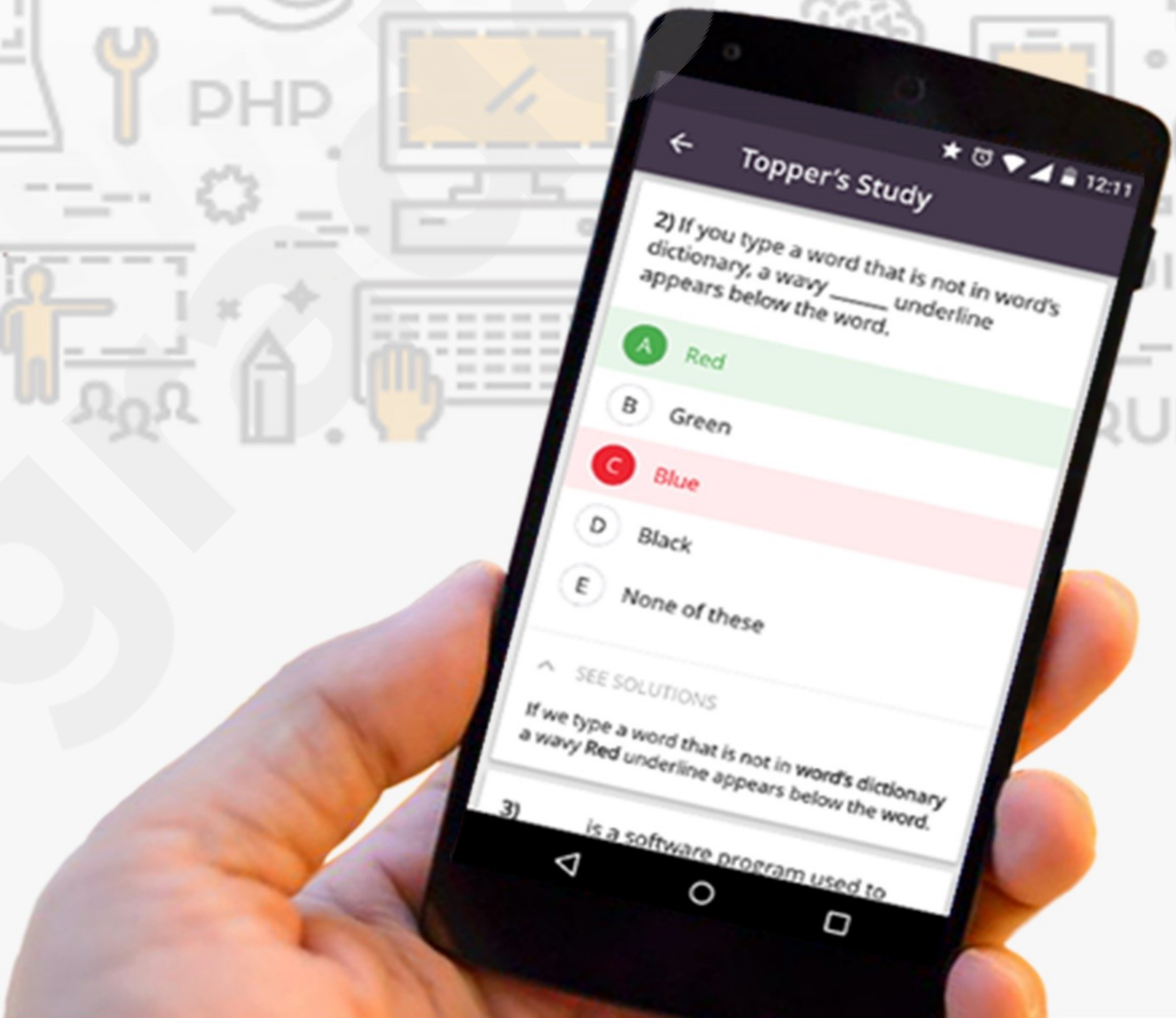
Data Structure

Questions & Solutions



Programming & Data Structures

1. Consider the C program fragment below which is meant to divide x by y using repeated subtractions. The variables x, y, q and r are all unsigned int.

```
(while (r >= y)
{
r = r – y;
q = q +1;
})
```

Which of the following conditions on the variables x, y, q and r before the execution of the fragment will ensure that the loop terminates in a state satisfying the condition

$x == (y * q + r)$?

A. (q = = r) && (r = =0)
B. (x > 0) && (r = =x) && (y > 0)
C. (q = = 0) && (r = = x) && (y > 0)
D. (q = = 0) && (y > 0)

2. Consider the following C program.

```
#include <stdio.h>
#include <string.h>
void printlength (char *s, char *t)
{
unsigned int c = 0;
int len = ((strlen (s) - strlen (t)) > c) ? strlen (s) : strlen (t);
  printf("%d\n", len);
}
void main()
{
  char *x = "abc";
  char *y = "defgh";
  printlength(x, y);
}
```

Recall that **strlen** is defined in **string.h** as returning a value of type **size_t**, which is an **unsigned int.** The output of the program is

A. 1
B. 2
C. 3
D. 4

Direction (3-4): *Consider the following C code segment.*

```
int a, b, c = 0;
void prtFun(void);
main( )
{ static int a = 1; /* Line 1 */
prtFun( );
a + = 1;
prtFun( );
printf (“ \n %d %d ”, a, b);
}
void prtFun(void)
{ static int a = 2; /* Line 2 */
int b = 1;
a + = ++b;
printf(“ \n %d %d ”, a, b);
} .
```

Programming & Data Structures

3. What output will be generated by the given code segment?

	3	1		4	2		4	2		3	1
A.	4	1	B.	6	1	C.	6	2	D.	5	2
	4	2		6	1		2	0		5	2

4. Consider the following C Program.

```
(# include <stdio.h>
#include< string.h>
#int main ( ) {
char* c = "GATECSIT2017";
char* p = c;
printf("%d", (int) strlen (c+p[2]-p[6]-1));
return 0;
})
```

The output of the program is _____.

A. 0 B. 1 C. 2 D. 3

5. Consider the following C program segment:

```
char p [20]
char * s = "string";
int length = strlen (s);
for (i = 0 ; i < length; i++)
p[i] = s[length - i];
printf("%s",p);
```

The output of the program is.

A. gnirts B. string C. gnirt D. no output is printed

6. Consider the following C program. The output of the program is ___________.

```
# include <stdio.h>
int f1(void);
int f2(void);
int f3(void);
int x = 10;
int main()
{
   int x = 1;
   x += f1() + f2() + f3() + f2();
   pirntf("%d", x);
   return 0;
}
int f1()
{
   int x = 25;
   x++;
   return x;
}
int f2( )
{
   static int x = 50;
   x++;
```

```
   return x;
}
int f3( )
{
   x *= 10;
   return x;
}
```

A. 230 B. 250 C. 260 D. 270

7. What does the following C-statement declare?

```
int (* f) (int *);
```

A. A function that takes an integer pointer as argument and returns an integer
B. A function that takes an integer as argument and returns an integer pointer
C. A pointer to a function that takes as integer pointer as argument and returns an integer
D. A function that takes an integer pointer as argument and returns a function pointer

8. Let T be a binary search tree with 15 nodes. The minimum and maximum possible heights of T are:
Note: The height of a tree with a single node is 0.
A. 4 and 15 respectively
B. 3 and 14 respectively
C. 4 and 14 respectively
D. 3 and 15 respectively

9. Consider the C code fragment given below.

```
typedef struct node
{
int data;
node* next ;
} node;
void join (node* m, node* n) {
node* p=n ;
while (p->next ! =NULL){
p = p -> next ;
}
p–> next = m;
}
```

Assuming that m and n point to valid NULL- terminated linked lists, invocation of join will
A. append list m to the end of list n for all inputs
B. either cause a null pointer dereference or append list m to the end of list n
C. cause a null pointer dereference for all inputs
D. append list n to the end of list m for all inputs

10. Consider the C functions foo and bar given below:

```
int foo (int val ) {
int x = 0;
while (val > 0) {
x = x + foo ( val --);
}
return val ;
}
int bar (int val ) {
int x = 0;
```

Programming & Data Structures

```
while (val > 0) {
x = x + bar (val – 1) ;
}
return val ;
}
```

Invocations of foo (3) and bar (3) will result in:

A. Return of 6 and 6 respectively.

B. Infinite loop and abnormal termination respectively.

C. Abnormal termination and infinite loop respectively.

D. Both terminating abnormally

11. Consider the C program below.

```
#include <stdio.h> int *A, stkTop;
int stkFunc(int opcode, int val)
{
static int size=0, stkTop=0;
switch (opcode) {
case -1: size = val; break;
case 0: if (stkTop < size) A[stkTop++] = val; break; default: if (stkTop) return A[--stkTop];
}
return -1;
}
int main()
{
int B[20]; A = B; stkTop = -1;
stkFunc (-1, 10);
stkFunc ( 0, 5);
stkFunc ( 0, 10);
printf ("%d\n", stkFunc(1, 0) + stkFunc(1,0));
}
```

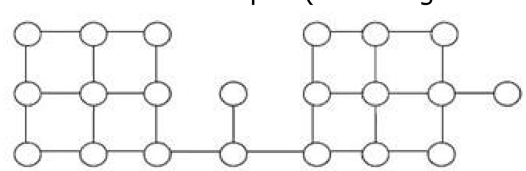
The value printed by the above program is ___________.

A. 5 B. 10 C. 15 D. 20

12. Let T be a tree with 10 vertices. The sum of the degrees of all the vertices in T is _____.

A. 0 B. 10 C. 18 D. 20

13. Suppose depth first search is executed on the graph below starting at some unknown vertex. Assume that a recursive call to visit a vertex is made only after first checking that the vertex has not been visited earlier. Then the maximum possible recursion depth (including the initial call) is _________.

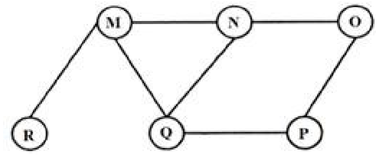


A. 17 B. 18

C. 19 D. 20

Programming & Data Structures

14. The Breadth First Search algorithm has been implemented using the queue data structure. One possible order of visiting the nodes of the following graph is



A. MNOPQR　B. NQMPOR　C. QMNPRO　D. QMNPOR

15. Suppose the elements 7, 2, 10 and 4 are inserted, in that order, into the valid 3- ary max heap found in the above question, Q.76. Which one of the following is the sequence of items in the array representing the resultant heap?

A. 10, 7, 9, 8, 3, 1, 5, 2, 6, 4
B. 10, 9, 8, 7, 6, 5, 4, 3, 2, 1
C. 10, 9, 4, 5, 7, 6, 8, 2, 1, 3
D. 10, 8, 6, 9, 7, 2, 3, 4, 1, 5

16. A complete binary min-heap is made by including each integer in [1, 1023] exactly once. The depth of a node in the heap is the length of the path from the root of the heap to that node. Thus, the root is at depth 0. The maximum depth at which integer 9 can appear is ____.

A. 6　B. 7　C. 8　D. 9

17. An operator delete (i) for a binary heap data structure is to be designed to delete the item in the i-th node. Assume that the heap is implemented in an array and i refers to the i-th index of the array. If the heap tree has depth d (number of edges on the path from the root to the farthest leaf), then what is the time complexity to re-fix the heap efficiently after the removal of the element?

A. O(1)
B. O(d) but not O(1)
C. O(2d) but not O(d)
D. O(d 2d) but not O(2d)

18. A priority queue is implemented as a Max-Heap. Initially, it has 5 elements. The level-order traversal of the heap is: 10, 8, 5, 3, 2. Two new elements 1 and 7 are inserted into the heap in that order. The level-order traversal of the heap after the insertion of the elements is:

A. 10, 8, 7, 3, 2, 1, 5
B. 10, 8, 7, 2, 3, 1, 5
C. 10, 8, 7, 1, 2, 3, 5
D. 10, 8, 7, 5, 3, 2, 1

19. You are given the post order traversal, P. of a binary search tree on then elements 1, 2, ..., 11. You have to determine the unique binary search tree that has P as its post-order traversal. What is the time complexity of the most efficient algorithm for doing this?

A. $\Theta(\log n)$

B. $\Theta(n)$

C. $\Theta(n \log n)$

D. none of the above, as the tree cannot be uniquely determined.

20. Consider the following New-order strategy for traversing a binary tree:

- Visit the root;
- Visit the right subtree using New-order;
- Visit the left subtree using New-order;

The New-order traversal of the expression tree corresponding to the reverse polish expression 3 4 * 5 − 2 ^ 6 7 * 1 + − is given by:

A. + - 1 6 7 * 2 ˆ 5 - 3 4 *
B. - + 1 * 6 7 ˆ 2 - 5 * 3 4
C. - + 1 * 7 6 ˆ 2 - 5 * 4 3
D. 1 7 6 * + 2 5 4 3 * - ˆ -

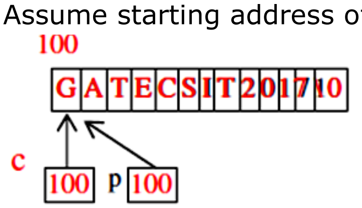
ANSWERS

1. Ans. C.
Given, program is:
(while (r y){
}
If we want to final value as Then initial value of r should be equal to x (Since y is subtracted from r each time in given code). q incremented by 1 (q is quotient here). To avoid undefined behavior, value of y should be greater than zero.
Therefore, (q == 0)&&(r == x)&&(y > 0))

2. Ans. C.
((strlen(s) – strlen(t)) > c) ? strlen (s) : strlen (t)
= (3 – 5 > 0)
= (-2 > 0)
Important point here is while comparing -2 with c, result will be a positive number as c is unsigned. So, out of these two, strlen (s) will be printed. Therefore, **option c** is correct

3. Ans. C.
main
a=1
prtFun()
a=2
b=1
a= a + ++b = 2+2 = 4
b = 2
printf --> 4 2
----
back to main
a = a+1 --> 1+1 -->2 (local static a is taken)
prtFun()
a=4 // previous value in the function is retained bcos of static
b=1
a= a + ++b = 4+2 = 6
b = 2
printf --> 6 2
----
back to main
a = 2
b = 0 (initial value of global b. in prtFun local b is only updated)
printf --> 2 0

4. Ans. C.
Assume starting address of string is 100 to make the whole expression is easy to understand.



ANSWERS

$$strlen(c+2[p]-6[p]-1)$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$100+T- \quad I-1$$

Note: Whenever we have characters in the arithmetic expressions, we can replace those with their ASCII values strlen

$$(100+x+11-x-1)$$

[assume x has the ASCII value of I]

$$\Rightarrow Strlen(110)$$

$$\therefore 2\ is\ printed$$

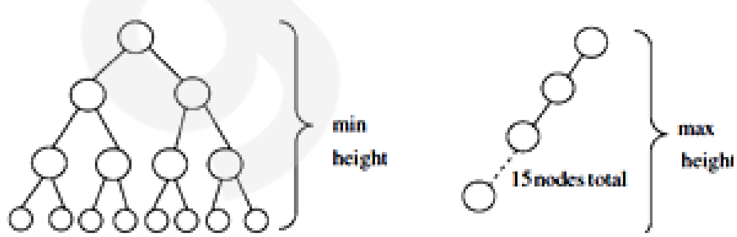
It gives address of second last character in the string. So it prints length 2.

5. Ans. D.
p [0] stores the null character \0
Printing the string from p address gives no output.

6. Ans. A.
x += f1() + f2() + f3() + f2();
x = x + f1() + f2() + f3() + f2();
f1() returns 26
f2() returns 51
f3() returns 100
second call to f2() returns 52
[Note x is static in f2()]
x = 1 + 26 + 51 + 100 + 52 = 230. The output of the program is 230

7. Ans. C.
A function that takes an integer pointer as argument and returns an integer -> int f (int *)
A function that takes an integer as argument and returns an integer pointer ->int * f (int)
A pointer to a function that takes an integer pointer as argument and returns an integer =>int (* f) (int *);

8. Ans. B.



Min height = floor($\log_2$ N) = floor(log 15)=3
Max height n-1 = 14, when the tree is either left skewed or right skewed.

9. Ans. B.
While loop in Join Procedure moves the pointer 'p' to the last node of the list "n". And at the last statement, we are initializing the next of the last node of list n to start of list "m".
But in some cases it may dereference to null pointer.

ANSWERS

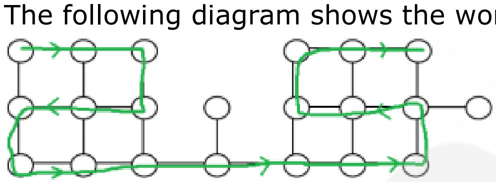
10. Ans. C.
In given function foo every time in the while foo is called value 3 because val is passed with post decrement operator so the value 3 is passed and val is decremented later. Every time the function is called a new variable is created as the passed variable is passed by value, with the value 3. So the function will close abruptly without returning any value.
In the function bar, in the while loop value the value of val variable is not decrementing, it remains 3 only. Bar function in the while loop is called with val-1 i.e 2 but value of val is not decremented, so it will result in infinite loop.

11. Ans. C.
The code in main, basically initializes a stack of size 10, then pushes 5, then pushes 10.
Finally the printf statement prints sum of two pop operations which is 10 + 5 = 15.

```
stkFunc (-1, 10); // Initialize size as 10
stkFunc (0, 5); // push 5
stkFunc (0, 10); // push 10
// print sum of two pop
printf ("%d\n", stkFunc(1,0)+stkFunc(1,0));
```

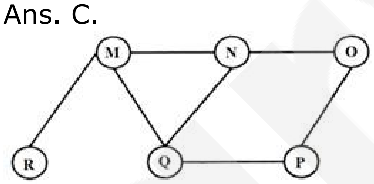
12. Ans. C.
Given, v= Total vertices = 10
e = v – 1 =9
Degree = 2 * e = 18
Therefore, option C is correct

13. Ans. C.
The following diagram shows the worst case situation where the recursion tree



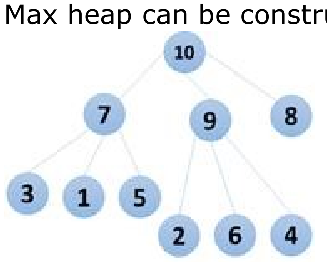
has maximum depth. So the recursion depth is 19 (including the first node).

14. Ans. C.



Q → M
Q → N
Q → P
Q → R
Q → O
Using BFS Algorithm Order of visiting the nodes QMNPRO

15. Ans. A.
Max heap can be constructed as,

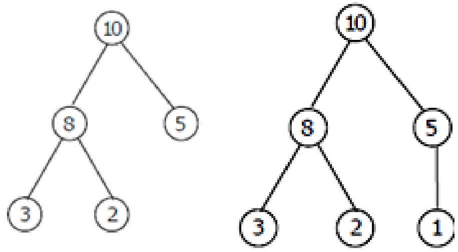


ANSWERS

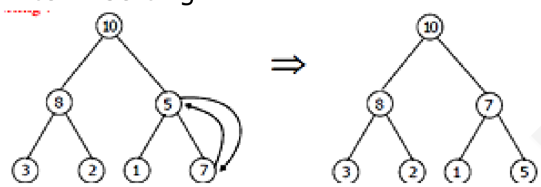
16. Ans. C.
n^{th} smallest element will be present within 'n' levels of min heap

17. Ans. B.
Time complexity of heapification is O (height) = O(d)

18. Ans. A.
Initial max-heap is after inserting 1



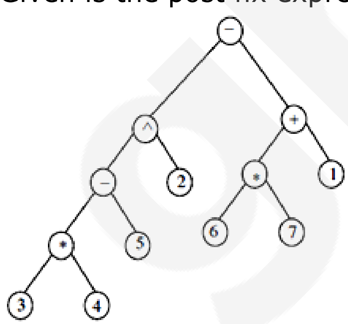
Heapification is not required as it satisfies max-heap property
After inserting 7



Hence level order traversal is 10,8,7,3,2,1,5

19. Ans. C.
Unique tree can be constructed only if in-order along with either pre-order or post-order traversal is known. in-order traversal of any BST is sorted order. From given Post-Order, we can compute inorder by sorting all elements. Since we know in-order and post-order of BST, so three can be constructed using O(n log n) Time.

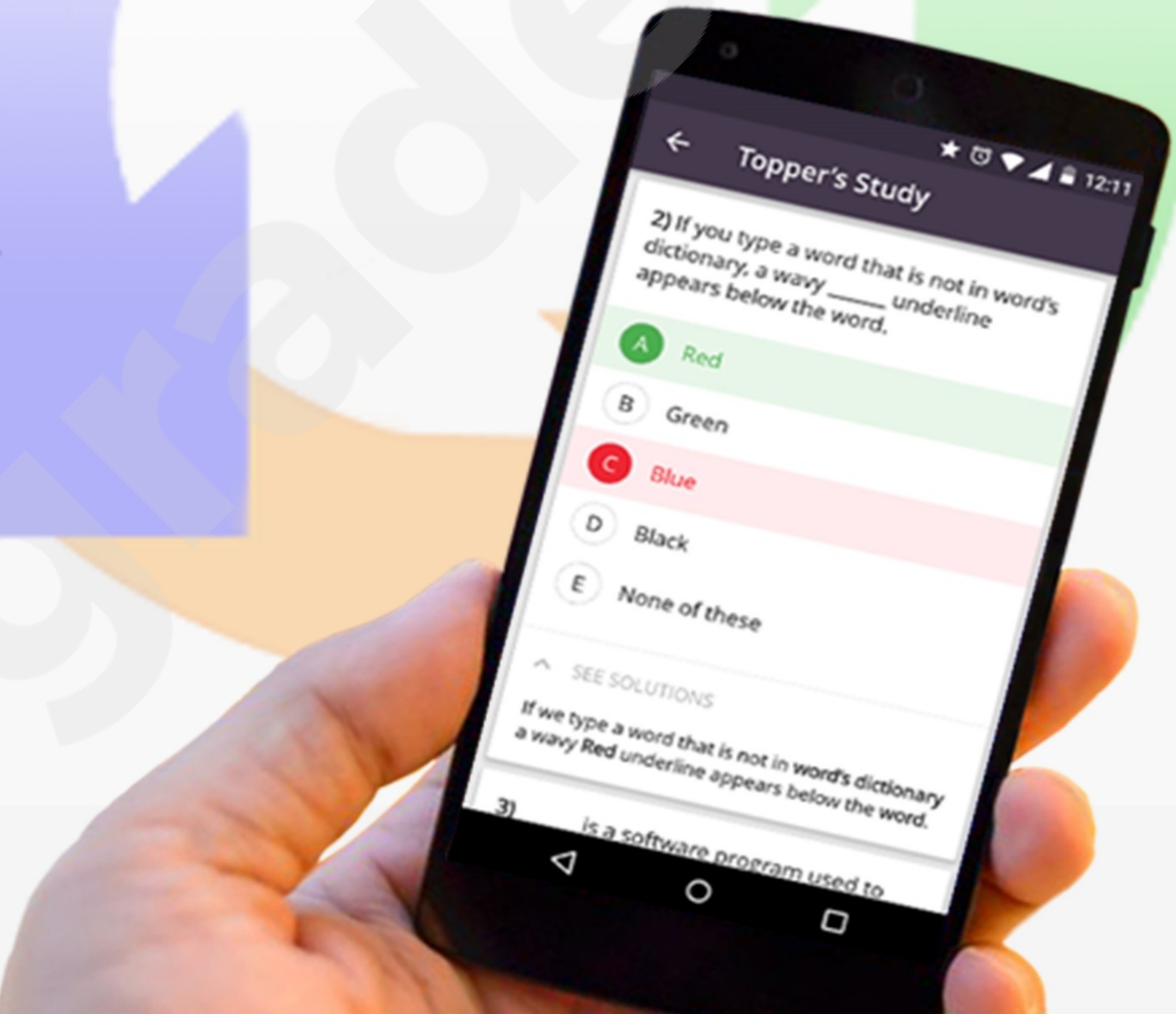
20. Ans. C.
Given is the post fix expression the expression tree given below.



New-order of above expression tree is
−+1*76∧2−5*43

Algorithms

Questions & Solutions



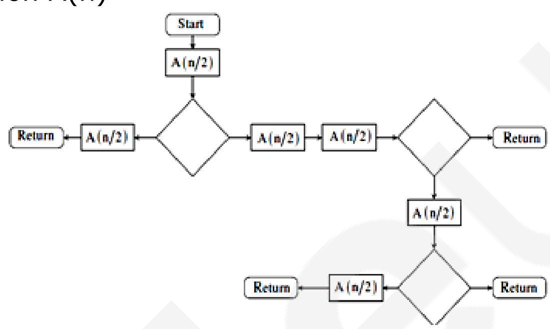
Algorithms

1. N items are stored in a sorted doubly linked list. For a delete operation, a pointer is provided to the record to be deleted. For a decrease-key operation, a pointer is provided to the record on which the operation is to be performed. An algorithm performs the following operations on the list in this order: $\Theta(N)$ delete, $O(\log N)$ insert, $O(\log N)$ find, and $\Theta(N)$ decrease-key. What is the time complexity of all these operations put together?

A. $O(\log^2 N)$ B. $O(N)$ C. $O(N^2)$ D. $\theta(N^2 \log N)$

2. The given diagram shows the flowchart for a recursive function A(n). Assume that all statements, except for the recursive calls, have O(1) time complexity. If the worst case time complexity of this function is $O(n^{\alpha})$, then the least possible value (accurate up to two decimal positions) of α is _____.

Flowchart for Recursive Function A(n)



A. 1.32 B. 2.32 C. 3.32 D. 4.32

3. Let $f(n) = n$ and $g(n) = n^{(1+\sin n)}$, where n is a positive integer. Which of the following statements is / are correct?

I. $f(n) = 0(g(n))$

II. $f(n) = \Omega(g(n))$

A. Only I B. Only II C. Both I and II D. Neither I nor II

4. An algorithm performs $(\log N)^{1/2}$ find operations, N insert operations, $(\log N)^{1/2}$ delete operations, and $(\log N)^{1/2}$ decrease-key operations on a set of data items with keys drawn from a linearly ordered set. For a delete operation, a pointer is provided to the record that must be deleted.

For the decrease-key operation, a pointer is provided to the record that has its key decreased. Which one of the following data structures is the most suited for the algorithm to use, if the goal is to achieve the best total asymptotic complexity considering all the operations?

A. Unsorted array B. Min-heap

C. Sorted array D. Sorted doubly linked list

5. Let G be a complete undirected graph on 4 vertices, having 6 edges with weights being 1, 2, 3, 4, 5, and 6. The maximum possible weight that a minimum weight spanning tree of G can have is ___________.

A. 4 B. 5 C. 6 D. 7

6. G = (V, E) is an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G. Which of the following statements about the minimum spanning trees (MSTs) of G is/are TRUE?

I. If e is the lightest edge of some cycle in G, then every MST of G includes e

II. If e is the heaviest edge of some cycle in G, then every MST of G excludes e

A. I only B. II only C. both I and II D. neither I nor II

Algorithms

7. The weight of a sequence $a_0, a_1, \ldots, a_{n-1}$ of real numbers is defined as $a_0 + a_1 / 2 + \ldots + a_{n-1}/2^{n-1}$. A subsequence of a sequence is obtained by deleting some elements from the sequence, keeping the order of the remaining elements the same. Let X denote the maximum possible weight of a subsequence of $a_0, a_1, \ldots, a_{n-1}$. Then X is equal to

A. $\max(Y, a_0 + Y)$ B. $\max(Y, a_0 + Y/2)$

C. $\max(Y, a_0 + 2Y)$ D. $a_0 + Y/2$

8. G is a graph on n vertices and $2n - 2$ edges. The edges of G can be partitioned into two edge-disjoint spanning trees. Which of the following is NOT true for G?

A. For every subset of k vertices, the induced sub-graph has at most $2k - 2$ edges

B. The minimum cut in G has at least two edges

C. There are two edge-disjoint paths between every pair of vertices

D. There are two vertex-disjoint paths between every pair of vertices

9. What is the minimum possible weight of a path P from vertex 1 to vertex 2 in this graph such that P contains at most 3 edges?

A. 7 B. 8 C. 9 D. 10

10. Consider the weighted undirected graph with 4 vertices, where the weight of edge {i, j} is given by the entry W_{ij} in the matrix W.

$$W = \begin{bmatrix} 0 & 2 & 8 & 5 \\ 2 & 0 & 5 & 8 \\ 8 & 5 & 0 & x \\ 5 & 8 & x & 0 \end{bmatrix}$$

The largest possible integer value of x, for which at least one shortest path between some pair of vertices will contain the edge with weight x is ___________.

A. 12 B. 13 C. 14 D. 15

11. An algorithm to find the length of the longest monotonically increasing sequence of numbers in an array A[0: *n* - 1] is given below.

Let L; denote the length of the longest monotonically increasing sequence starting at index *i* in the array.

Initialize $L_{n-1} = 1$.

For all i such that $0 \le i \le n - 2$

$$L_1 = \begin{cases} 1 + L_{i+1} & \text{if } A[i] < A[i+1] \\ 1 & \text{otherwise} \end{cases}$$

Finally the length of the longest monotonically increasing sequence is Max $(L_0, L_1, \ldots\ldots, L_{n-1})$.

Which of the following statements is **TRUE**?

A. The algorithm uses dynamic programming paradigm

B. The algorithm has a linear complexity and uses branch and bound paradigm

C. The algorithm has a non-linear polynomial complexity and uses branch and bound paradigm

D. The algorithm uses divide and conquer paradigm.

12. Two alternative packages A and B are available for processing a database having 10^k records. Package A requires $0.0001n^2$ time units and package B requires $10n\log_{10}n$ time units to process n records. What is the smallest value of k for which package B will be preferred over A?

A. 12 B. 10 C. 6 D. 5

Algorithms

13. Consider an undirected unweighted graph G. Let a breadth-first traversal of G be done starting from a node r. Let d(r,u) and d(r,v) be the lengths of the shortest paths from r to u and v respectively in G. If u is visited before v during the breadth-first traversal, which of the following statements is correct?

A. d(r,u)<d(r,v)
B. d(r,u)>d(r,v)
C. d(r,u)≤d(r,v)
D. None of the above

Direction (14-15): The subset-sum problem is defined as follows. Given a set of n positive integers, S = {a_1, a_2, a_3 ………, a_n}, and a positive integer W, is there a subset of *S* whose elements sum to TV? A dynamic program for solving this problem uses a 2 - dimensional Boolean array, *X*, with n rows and *W + 1* columns. $X[i, j], 1 \leq i \leq n,$ $0 \leq j$ *W*, is TRUE if and only if there is a subset of {$a_1, a_2,$ ………, a_i} whose elements sum to J.

14. Which of the following is valid for 2 ≤ i ≤ n and $a_1 \leq j \leq W$?

A. X[i, j] = X[i-1, j] v X[i, j-a_1]
B. X[i, j] = X[i-1, j] v X[i-1, j-a_1]
C. X[i, j] = X[i-1, j] ∧X[i, j-a_1]
D. X[i, j] = X[i-1, j] ∧X[i-1, j-a_1]

15. Given two arrays of numbers $a_1, …, a_n$ and $b_1, …, b_n$ where each number is 0 or 1, the fastest algorithm to find the largest span (i, j) such that $a_i + a_{i+1} + …. + a_j = b_i + b_{i+1} + … + b_j$, or report that there is not such span

A. Takes $O(3^n)$ and $\Omega(2^n)$ time if hashing is permitted
B. Takes $O(n^3)$ and $\Omega(n^{2.5})$ time in the key comparison model
C. Takes $\Theta(n)$ time and space
D. Tales $O(\sqrt{n})$ time only if the sum of the 2n elements is an even number

16. Consider the recurrence function

$$T(n)=\begin{cases}2T(\sqrt{n})+1, n>2\\ 2, \quad 0<n\leq 2\end{cases}$$

Then T(n) in terms of θ notation is

A. $\theta(\log\log n)$
B. $\theta(\log n)$
C. $\theta(\sqrt{n})$
D. $\theta(n)$

17. A message is made up entirely of characters from the set X= {P, Q, R, S, T}. The table of probabilities for each of the characters is shown below:

Character	Probability
P	0.22
Q	0.34
R	0.17
S	0.19
T	0.08
Total	1.00

If a message of 100 characters over X is encoded using Huffman coding, then the expected length of the encoded message in bits is______.

A. 225
B. 232
C. 245
D. 255

Algorithms

18. Consider the following functions from positive integers to real numbers:

$$10, \sqrt{n}, n, \log_2 n, \frac{100}{n}$$

The CORRECT arrangement of the above functions in increasing order of asymptotic complexity is:

A. $\log_2 n, \frac{100}{n}, 10, \sqrt{n}, n$

B. $\frac{100}{n}, 10, \log_2 n, \sqrt{n}, n$

C. $10, \frac{100}{n} \sqrt{n}, \log_2 n, n$

D. $\frac{100}{n}, \log_2 n, 10, \sqrt{n}, n$

19. Let A be an array of 31 numbers consisting of sequence of 0's followed by a sequence of 1's. The problem is to find the smallest index i that A[i] is 1 by probing the minimum numbers of locations in A. The worst case number of probes performed by an optimal algorithm is _____.

A. 2

B. 3

C. 4

D. 5

20. G = (V, E) is an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G. Which of the following statements about the minimum spanning trees (MSTs) of G is/are TRUE?

I. If e is the lightest edge of some cycle in G, then every MST of G includes e

II. If e is the heaviest edge of some cycle in G, then every MST of G excludes e

A. I only

B. II only

C. both I and II

D. neither I nor II

ANSWERS

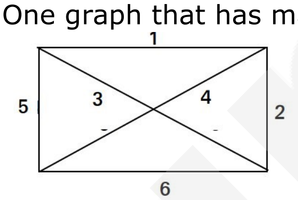
1. Ans. C.
Delete: θ(1) time as pointer is directly given.
Insert: O(N) time, we may need to insert at the end of the sorted list.
Find: θ(N) time. List we need to search sequentially.
Decrease key: θ(N) time, pointer is directly given, delete then insert.
All operations when put together:
$\theta(N) * \theta(1) + O(\log N) * O(N) + O(\log N) * \theta(N) + \theta(N) * \theta(N)$
This is $O(N^2)$.

2. Ans. B.
If they are asking for worst case complexity hence,
By calling A(n) we get A(n/2) 5 times,
A(n)=5A(n/2)+O(1)
Hence by applying masters theorem,
Case 1 : $a>b^k$
$n^\wedge(\log_2 5)$
Thus value of alpha will be 2.32

3. Ans. D.
The value of sin function varies from -1 to 1.
For sin = -1 or any other negative value,
I become false.
For sin = 1 or any other negative value,
II becomes false, Result is Option D.

4. Ans. A.
The time complexity of insert in unsorted array is O(1), O(Logn) in Min-Heap, O(n) in sorted array and sorted DLL. Since number of insertion operations is asymptotically higher, unsorted array is preferred.

5. Ans. D.
One graph that has maximum possible weight of spanning tree



6. Ans. B.
Statement II is correct. If e is the heaviest edge in cycle every mst excludes it.
Statement 1 is incorrect. Complete graph with 4 vertices and edge weights 1,2,5,6 in non diagonal and diagonal edges 3 and 4. 4,5,6 will create a cycle and we can exclude the lighest edge e (4) from it, in a MST. So every MST of G need not include e.

7. Ans. B.
Using concepts of Dynamic Programming, to find the maximum possible weight of a subsequence of X, we will have two alternatives
1. Do not include a0 in the subsequence: then the maximum possible weight will be equal to maximum possible weight of a subsequence of {a1, a2, …..an} which is represented by Y
2. Include a0: then maximum possible weight will be equal to a0 + (each number picked in Y will get divided by 2) <=> a0 + Y/2. Here you can note that Y will itself pick optimal subsequence to maximize the weight.
Final answer will be Max (Case1, Case2) i.e. Max (Y, a_0 + Y/2). Hence B).

ANSWERS

8. Ans. C.

A graph G having n vertices and 2n-2 edges, the edges of G can be partitioned into two edges disjoint spanning trees. There would not be two edges-disjoint paths between every pair of vertices.

9. Ans. B.

From going $1 \rightarrow 2$

$V_1 \xrightarrow{1} V_0 \xrightarrow{4} V_4 \xrightarrow{3} V_3$

So, total weight = 8

Hence it contains 3 edges.

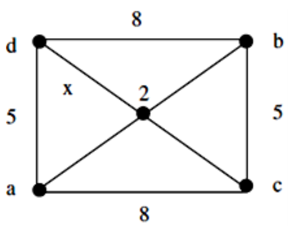
[Here weight is given in matrix. We use directly such that

$w_s j_i = V_i \rightarrow V_i j$

$V_1 V_0 \rightarrow V_0 V_4 \rightarrow V_4 V_2$

i.e. 1 + 4 + 3 = 8

10. Ans. A.



If $x = 12$ then the shortest path between d & c will contain edge with lable 'x'.

11. Ans. A.

We set the L_{n-1} initially and other values L_k are computed in backwards order starting from k = n – 2 through k = 0. Each value L_k depends on the L_{k+1} which is already computed and hence uses dynamic programming. The algorithm has a linear complexity, but it does not use branch and bound paradigm.

12. Ans. C.

B must be preferred on A as

$0.0001\, n^2 < 10 n \log_{10} n$

$\Rightarrow 10^{-5} n < \log_{10} n$

$\Rightarrow 10^5 < \log_{10} n^{1/n}$

$\Rightarrow n 10^{-5} < \log_{10} n$

We know

$\log_{10} n = k$

$\Rightarrow 10^k = t$

$\therefore k > 10^{-5} 10k$

$\Rightarrow k > 10^{k-5}$

$\Rightarrow k - 5 > 0$

$\Rightarrow k > 5$

then min k = 6

ANSWERS

13. Ans. C.

In an undirected graph, there can be maximum n(n-1)/2 edges. We can choose to have (or not have) any of the n(n-1)/2 edges. So, total number of undirected graphs with n vertices is 2^(n(n-1)/2).

An undirected graph is graph, i.e., a set of objects (called vertices or nodes) that are connected together, where all the edges are bidirectional. An undirected graph is sometimes called an undirected network. In contrast, a graph where the edges point in a direction is called a directed graph.

14. Ans. B.

$X[i, j] = X[i-1, j] \vee X[i-1, j-a_1]$

15. Ans. C.

Since array is binary, the max sum will go until n and so the sum difference of the two arrays can vary between –n and n. We use array start to keep the starting index of each possible sum (hence of size 2n+1) and array end to keep the ending index. So, our required solution will be max(end[i]-start[i]) provided both are assigned values.

Program will be as;

```
#include <stdio.h>
#define size 100 //assume n is less than 100
int main()
{
int n, a[size], b[size];
int start[2*size+1], end[2*size+1];
int sum1 = 0, sum2 = 0, i;
int diff[size];
printf("Enter n: ");
scanf("%d", &n);
for(i = 0; i < n; i++)
{
printf("Enter a[%d]: ", i);
scanf("%d", &a[i]);
}
for(i = 0; i < n; i++)
{
printf("Enter b[%d]: ", i);
scanf("%d", &b[i]);
}
for(i = 0; i < n; i++)
{
if(a[i]) sum1++;
if(b[i]) sum2++;
diff[i] = sum1 - sum2;
}
for(i = 0; i < 2*n; i++)
start[i] = -1,end[i] = -1;
start[n] = end[n] = 0;
//initially sum is 0 at the beginning of array and
//the first n-1 elements of start and end are used
```

ANSWERS

```
//if sum of A till ith element is less than sum of B till ith element
for(i=0; i < n; i++)
{
if(start[diff[i] + n] == -1)//interested only in the first occurrence of diff[i]
start[diff[i] + n] = i;
end[diff[i] + n] = i;//interested in the last occurrence of diff[i]
}
int max = -1;
int savei = -1; //savei is for storing the sum having the largest span
for(i = 0; i < 2*n; i++)
{
if(start[i] > -1 && (end[i] - start[i] > max))
{
max = end[i] - start[i];
savei = i;
}
}
if(savei >= 0)
{
printf("The largest span is from %d to %d\n", start[savei]+(savei != n), end[savei]);
//when sum zero is having the largest span, span starts from first element itself.
//Else, the span starts from the next element from which the span does not change
}
else
{
printf("No span\n");
}
}
```

16. Ans. B.

$$T(n) = 2T\left(\sqrt{n}\right) + 1$$

Put

$$n = 2^K$$

$$T\left(2^K\right) = 2T\left(2^{K/2}\right) + 1$$

Assume

$$T\left(2^k\right) = \delta(K)$$

$$\Rightarrow \ \delta(K) = 2\delta\left(\frac{K}{2}\right) + 1$$

By master's theorem

$$\delta(K) = \theta(K)$$

$$T\left(2^k\right) = \theta(K)$$

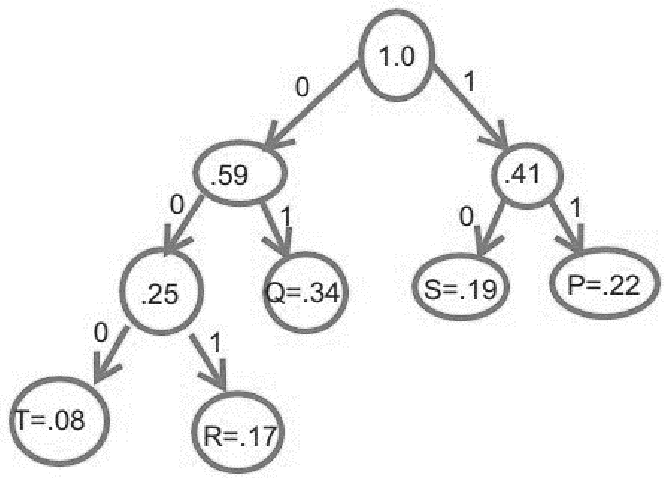
$$T(n) = \theta(\log n) \quad \because 2^k = n$$

ANSWERS

17. Ans. A.

Huffman tree is as follows

T=.08 R=.17 S=.19 P=.22 Q=.34



no of bit required for each alphbet:
T=3bit, R=3bit, Q=2bit, S=2bit, P=2bit
agv length/char=no of bit* frequency of occurance of each alphabet
=3*.08+3*.17+2*.34+2*.19+2*.22=2.25bits
so for 100 char=2.25*100=225bits

18. Ans. B.

functions in increasing order of asymptotic complexity is:

$$\frac{100}{n}, 10, \log_2 n, \sqrt{n}, n$$

(Hint : Put large values of n to verify)

19. Ans. D.

In the given array the elements are 0's followed by 1's, which means array is already sorted. So, we can apply binary search. At each stage, we compare $A\left[\frac{low+high}{2}\right]$.

[Assuming 'A' is an array of 31 elements] with '1' and if it is 1 we check the left part recursively and if it is '0' we check the right part of the array recursively, which takes log2 31 comparisons in the worst case.

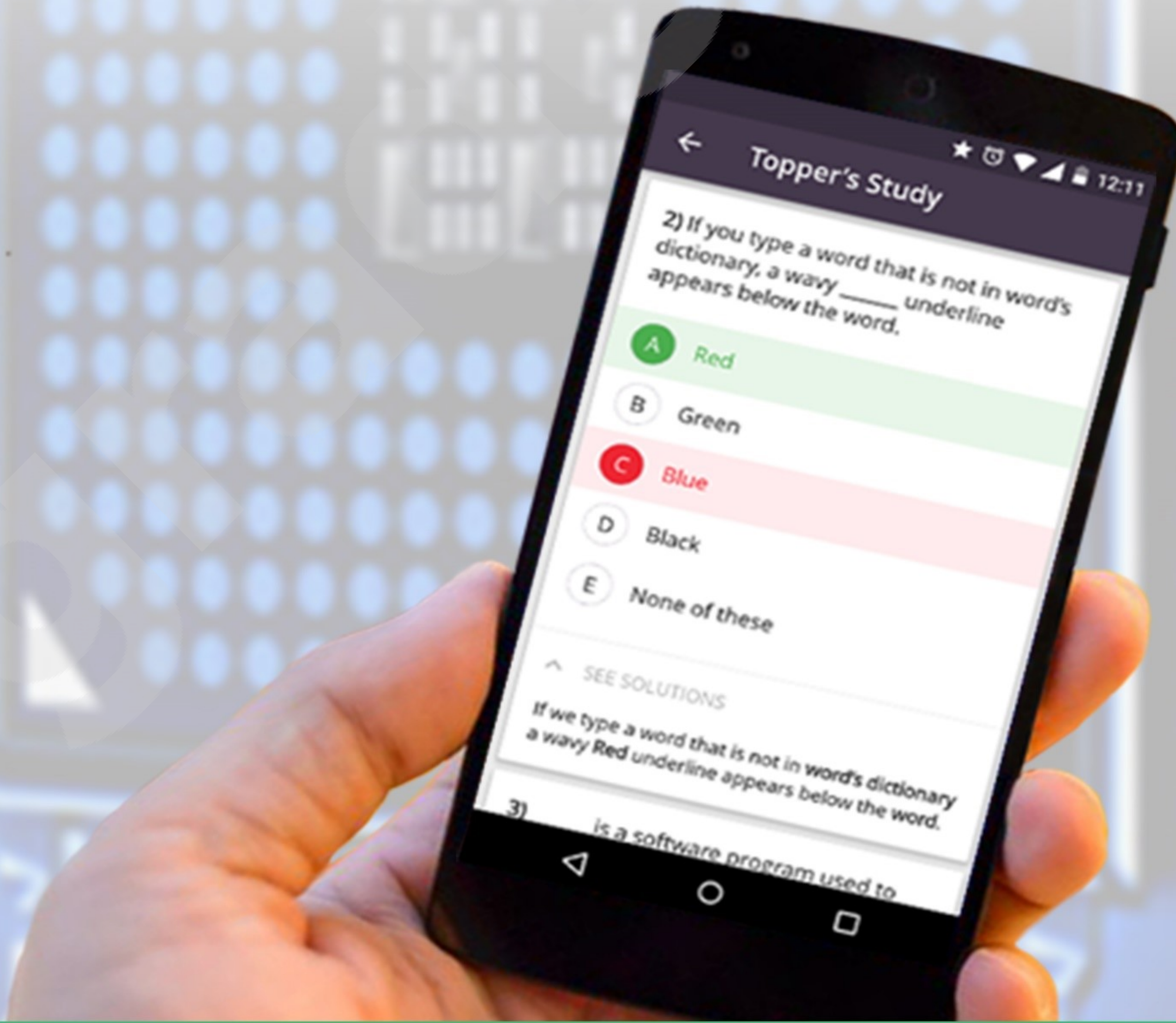
20. Ans. B.

Statement II is correct. If e is the heaviest edge in cycle every mst excludes it.

Statement 1 is incorrect. Complete graph with 4 vertices and edge weights 1,2,5,6 in non diagonal and diagonal edges 3 and 4. 4,5,6 will create a cycle and we can exclude the lighest edge e (4) from it, in a MST. So every MST of G need not include e.

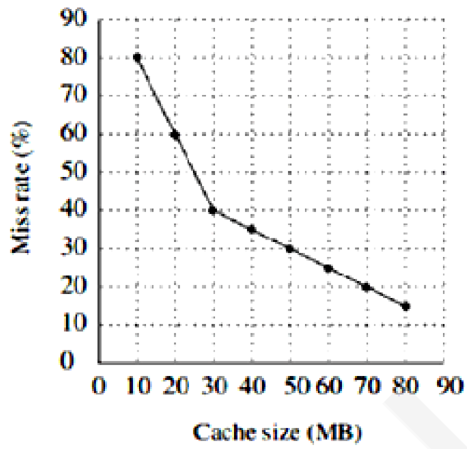
gradeup

Computer Organization
Questions & Solutions



Computer Organizations

1. A file system uses an in-memory cache to cache disk blocks. The miss rate of the cache is shown in the figure. The latency to read block from the cache is 1ms and to read a block from the disk is 10 ms. Assume that the cost of checking whether a block exists in the cache is negligible. Available cache sizes are in multiples of 10 MB.



The smallest cache size required to ensure an average read latency of less than 6 ms is ____ MB.

A. 30 B. 40 C. 50 D. 60

2. When there is a miss in both L1 cache and L2 cache, first a block is transferred from main memory to L2 cache, and then a block is transferred from L2 cache to L1 cache. What is the total time taken for these transfers ?

A. 222 nanoseconds B. 888 nanoseconds
C. 902 nanoseconds D. 968 nanoseconds

Direction (3-4): A computer system has an L1 cache, an L2 cache, and a main memory unit connected as shown below. The block size in L1 cache is 4 words. The block size in L2 cache is 16 words. The memory access times are 2 nanoseconds. 20 nanoseconds and 200 nanoseconds for L1 cache, L2 cache and main memory unit respectively.

3. When there is a miss in L1 cache and a hit in L2 cache, a block is transferred from L2 cache to L1 cache. What is the time taken for this transfer?

A. 2 nanoseconds B. 20 nanoseconds
C. 22 nanoseconds D. 88 nanoseconds

4. Which of the following array elements has the same cache index as ARR [0] [0]?

A. ARR [0] [4] B. ARR [4] [0]
C. ARR [0] [5] D. ARR [5] [0]

5. Which of the following lines of the data cache will be replaced by new blocks in accessing the array for the second time?

A. Line 4 to line 11 B. Line 4 to line 12
C. Line 0 to line 7 D. Line 0 to line 8

Computer Organizations

6. Consider a machine with a byte addressable main memory of 2^{16} bytes. Assume that directs mapped data cache consisting of 32 lines of 64 bytes each is used in the system. A 50 × 50 two-dimensional array of bytes is stored in the main memory starting from memory location 1100H. Assume that the data cache is initially empty. The complete array is accessed twice. Assume that the contents of the data cache do not change in between the two accesses.
How many data cache misses will occur in total?
A. 48 B. 50 C. 56 D. 59

7. Consider two cache organizations: The first one is 32 KB 2-way set associative with 32- byte block size. The second one is of the same size but direct mapped. The size of an address is 32 bits in both cases. A 2-to-1 multiplexer has a latency of 0.6 ns while a k bit comparator has a latency of k 10 ns. The hit latency of the set associative organization is $h_1$while that of the direct mapped one is h_2.
The value of $h_1$is:
A. 2.4 ns B. 2.3 ns C. 1.8 ns D. 1.7 ns

8. Consider a 2-way set associative cache with 256 blocks and uses LRU replacement, Initially the cache is empty. Conflict misses are those misses which occur due the contention of multiple blocks for the same cache set. Compulsory misses occur due to first time access to the block. The following sequence of accesses to memory blocks.
(0,128,256,128,0,128,256,128,1,129,257,129,1,129,257,129) is repeated 10 times. The number of conflict misses experienced by the cache is __________.
A. 43 B. 56 C. 63 D. 76

9. Consider the following code sequence having five instructions $I_1$to I_5. Each of these instructions has the following format.
OPRi, Rj, Rk
where operation OP is performed on contents of registers Rj and Rk and the result is stored in register Ri.
I_1:ADDR1, R2, R3
I_2:MULR7, R1, R3
I_3:SUBR4, R1, R5
I_4:ADDR3, R2, R4
I_5:MULR7, R8, R9
Consider the following three statements.
S1: There is a nanti-dependence between instructions $I_2$and I_5
S2: There is a nanti-dependence between instructions I_2 and I_4
S3:With in an instruction pipeline a nanti-dependence always creates one or more stalls
Which one of above statements is / are correct?
A. OnlyS1istrue B. OnlyS2istrue
C. OnlyS1andS3aretrue D. OnlyS2andS3aretrue

10. Suppose the functions F and G can be computed in 5 and 3 nanoseconds by functional units U_F and U_G respectively. Given two instances of U_F and two instances of U_G it is required to implement the computation $F(G(X_i))$ for $1 \le i \le 10$. Ignoring all other delays, the minimum time required to complete this computation is ____ nanoseconds.
A. 18 B. 28 C. 38 D. 48

11. What is the maximum percentage of time the CPU is held up for this disk I/O for cycle-stealing DMA transfer?
A. 80% B. 50% C. 7.5% D. none of the above

Computer Organizations

12. Consider a new instruction named branch-on-bit-set (mnemonic bbs). The instruction "bbs reg, pos, label" jumps to label if bit in position pos of register operand reg is one. A register is 32 bits wide and the bits are numbered 0 to 31, bit in position 0 being the least significant. Consider the following emulation of this instruction on a processor that does not have bbs implemented. temp¬reg & mask Branch to label if temp is non-zero. The variable temp is a temporary register. For correct emulation, the variable mask must be generated by:
A. mask ← 0 × 1 pos
B. mask ← 0 × ffffffffpos
C. mask ← pos
D. mask ← 0 ×f

13. Consider the following assembly language program for a hypothetical processor A, B, and C are 8 bit registers. The meanings of various instructions are shown as comments.
MOV B, #0 ; B ← 0
MOV C, #8 ; C ← 8
Z: CMP C, #0 ; compare C with 0
JZ X ; jump to X if zero flag is set
SUB C, #1 ; C ← C - 1
RRC A, #1 ; right rotate A through carry by one bit. Thus:
; if the initial values of A and the carry flag are $a_7..a_0$and
; c_0 respectively, their values after the execution of this
; instruction will be $c_0a_7..a_1$ and a_0 respectively.
JC Y ; jump to Y if carry flag is set
JMP Z ; jump to Z
Y: ADD B, #1 ; B ← B + 1
JMP Z ; jump to Z
X: .
If the initial value of register A is A_0 the value of register B after the program execution will be
A. the number of 0 bits in A_0
B. the number of 1 bits in A_0
C. A_0
D. 8

14. Consider the following program segment for a hypothetical CPU having three user registers R1, R2 and R3.

Instruction	*Operation*	*Instruction Size (in words)*
MOV R1,5000	;R1 ← Memory[5000]	2
MOV R2(R1)	;R2 ← Memory[(R1)]	1
ADD R2, R3	;R2 ← R2 + R3	1
MOV 6000, R2	; Memory[6000] ← R2	2
HALT	;Machine halts	1

Consider that the memory is byte addressable with size 32 bits, and the program has been loaded starting form memory location 1000 (decimal). If an interrupt occurs while the CPU has been halted after executing the HALT instruction, the return address (in decimal) saved in the stack will be
A. 1007
B. 1020
C. 1024
D. 1028

Direction (15-17): Consider the following program segment. Here R_1, R_2 and R_3 are the general purpose registers.

Instruction (no.of words)	Operation	Instruction size	
MOV R1, (3000)	R1←m[3000]	2	
LOOP:	MOV R2, (R3)	R2←M[R3]	1
ADD R2, R1	R2←R1 + R2	1	
MOV (R3), R2	M[R3]←R2	1	
INC R3	R3←R3 +1	1	
DEC R1	R1←R1 – 1	1	
BNZ LOOP	Branch on not zero	2	
HALT	Stop	1	

Assume that the content of memory location 3000 is 10 and the content of the register R3 is 2000. The content of each of the memory locations from 2000 to 2010 is 100. The program is loaded from the memory location 1000. All the numbers are in decimal.

Computer Organizations

15. Assume that the memory is word addressable. The number of memory references for accessing the data in executing the program completely is:
A. 10 B. 11 C. 20 D. 21

16. A CPU has a five-stage pipeline and runs at 1 GHz frequency. Instruction fetch happens in the first stage of the pipeline. A conditional branch instruction computes the target address and evaluates the condition in the third stage of the pipeline. The processor stops fetching new instructions following a conditional branch until the branch outcome is known. A program executes 10^9 instructions out of which 20% are conditional branches. If each instruction takes one cycle to complete on average, the total execution time of the program is:
A. 1.0 second B. 1.2 seconds C. 1.4 seconds D. 1.6 seconds

17. The following code is to run on a pipelined processor with one branch delay slot:
I_1: ADD $R_2 \leftarrow R_7 + R_8$
I_2: SUB $R_4 \leftarrow R_5 - R_6$
I_3: ADD $R_1 \leftarrow R_2 + R_3$
I_4: STORE Memory [R_4] $\leftarrow R_1$
BRANCH to Label if $R_1 == 0$
Which of the instructions I_1, I_2, I_3 or I_4 can legitimately occupy the delay slot without any other program modification?
A. I_1 B. I_2 C. I_3 D. I_4

18. A 5-stage pipelined processor has Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO) and Write Operand (WO) stages. The IF, ID, OF and WO stages take 1 clock cycle each for any instruction. The PO stage takes 1 clock cycle for ADD and SUB instructions, 3 clock cycles for MUL instruction, and 6 clock cycles for DIV instruction respectively. Operand forwarding is used in the pipeline. What is the number of clock cycles needed to execute the following sequence of instructions?
I_0 : MUL R_2, R_0, R_1 ; $R_2 \leftarrow R_0 * R_1$
I_1 : DIV R_5, R_3, R_4 ; $R_5 \leftarrow R_3/R_4$
I_2 : ADD R_2,R_5, R_2 ; $R_2 \leftarrow R_5 + R_2$
I_3 : SUB R_5, R_2, R_6 ; $R_5 \leftarrow R_2 - R_6$
A. 13 B. 15 C. 17 D. 19

19. Consider an instruction pipeline with five stages without any branch prediction: Fetch Instruction (FI), Decode Instruction (DI), Fetch Operand (FO), Execute Instruction (EI) and Write Operand (WO). The stage delays for FI, DI, FO, EI and WO are 5 ns, 7 ns, 10 ns, 8 ns and 6 ns, respectively. There are intermediate storage buffers after each stage and the delay of each buffer is 1 ns. A program consisting of 12 instructions I_1, I_2, I_3, …, I_{12} is executed in this pipelined processor. Instruction I4 is the only branch instruction and its branch target is I9. If the branch is taken during the execution of this program, the time (in ns) needed to complete the program is
A. 132 B. 165 C. 176 D. 328

20. An instruction pipeline has five stages, namely, instruction fetch (IF), instruction decode and register fetch (ID/RF), instruction execution (EX), memory access (MEM), and register writeback (WB) with stage latencies 1 ns, 2.2 ns, 2 ns, 1 ns, and 0.75 ns, respectively (ns stands for nanoseconds). To gain in terms of frequency, the designers have decided to split the ID/RF stage into three stages (ID, RF1, RF2) each of latency 2.2/3 ns. Also, the EX stage is split into two stages (EX1, EX2) each of latency 1 ns. The new design has a total of eight pipeline stages. A program has 20% branch instructions which execute in the EX stage and produce the next instruction pointer at the end of the EX stage in the old design and at the end of the EX2 stage in the new design. The IF stage stalls after fetching a branch instruction until the next instruction pointer is computed. All instructions other than the branch instruction have an average CPI of one in both the designs. The execution times of this program on the old and the new design are P and Q nanoseconds, respectively. The value of P/Q is __________.
A. 1.5 B. 1.4 C. 1.8 D. 2.3

ANSWERS

1. Ans. A.
   Look aside Cache Latency = 1ms
   Main Memory Latency = 10ms
   (1) Assume cache size is 20 MB:
   Miss rate = 60% ,
   Hit rate = 40%
   Average =0.4 (1) +0.6 (10) = 0.4 +6 = 6.4 ms > 6 ms
   (2) Assume cache size is 30 MB:
   Miss rate = 40% ,
   Hit rate = 60%
   Average = 0.6 (1) + 0.4 (10) = 0.6 + 4 = 4.6 ms < 6ms
   So Required smallest cache size is 30 MB to ensure an average read latency of less than 6ms.

2. Ans. D.
   From main memory, now again time of access is 200 n second. So access means for L2 came from main memory.
   Total time= block transfer time from smin memory to cache L2 + access time of L2 cache+ access time of L1 Cache= 8*20+ 4*200+ 2*4=968 sec

3. Ans. C.
   A block is transferred from L2 to L1. And L1 block size being 4 words (since L1 is requesting we need to consider L1 block size and not L2 block size) and data width being 4 bytes, it requires one L2 access (for read) and one L1 access (for store). So, time = 20+2 = 22 ns.

4. Ans. B.
   ARR[4] [0] and ARR [0][0] has the same cache index
   Number of elements that fit in a page = 4 KB/8 B = V elements
   Number of elements in any row of the array = 1024 = 210 elements
   Number of words in an array element = 8 B/1 B = 8 words.
   Two main memory addresses will be mapped to the same cache set if they have the same bit pattern for the Set Index bits.
   The address where array element arr[0] [0] is stored is 0xff000 or 0x000ff000 since the address is a 32bit address [8 × 4].
   0x000FF000 = 00000000000011111111000000000000 - (1)
   0x00107000 = 00000000000100000111000000000000 - (2)

0x000FF000	0	ARR[0][0]
	1	ARR[0][0]
	2	ARR[0][0]
	3	ARR[0][0]
	4	ARR[0][0]
	5	ARR[0][0]
	6	ARR[0][0]
	7	ARR[0][0]
0x000FF009	8	ARR[0][1]
	9	ARR[0][1]
	10	ARR[0][1]
	11	ARR[0][1]
	12	ARR[0][1]
	13	ARR[0][1]
	14	ARR[0][1]
0x000FF00F	15	ARR[0][1]

ANSWERS

0x000FF010	0	ARR[0][2]
	1	ARR[0][2]
	2	ARR[0][2]
	3	ARR[0][2]
	4	ARR[0][2]
	5	ARR[0][2]
	6	ARR[0][2]
	7	ARR[0][2]
0x000FF019	8	ARR[0][3]
	9	ARR[0][3]
	10	ARR[0][3]
	11	ARR[0][3]
	12	ARR[0][3]
	13	ARR[0][3]
	14	ARR[0][3]
0x000FF01F	15	ARR[0][3]

Addresses OxOOOFFOOO - OxOOOFFOOF have the same Set Index bits. There are sixteen addresses in this range each corresponding to one word. Since each array element takes up 8 words, this address range can store only two elements, ARR[0] [0] & ARR[0] [1] . So these two elements map to one of the 2 ways [each way is one cache block = 16 B] of some set k in the cache.

The next address for which the Set Index bits are the same is (2)

The number of addresses/words between these (1) & (2) = 2(11+4).

The number of array elements that can be stored in this address range

= 215/8 = 212 = 22 × 210

* 4 rows can be stored in this address range. So the last address, 0x00107000, will store ARR[4] [0] .

So the first element of every 4th row will map to the same cache set.

5. Ans. C.

No of page faults = 452

One line has 64 B

So the line at which these page faults will finish.

$$= \frac{452}{64}, 7$$

So 0 to 7 line

Hence (C) is correct option.

6. Ans. C.

Size of main memory 2^{16} bytes

Size of Cache = 32 × 64 B

= 2^{11} B Size of array = 2500 B

Array is stored in main memory but cache is empty.

Size of Cache = 2048 B

So no. of page faults = 2500 - 2048 = 452

For second access = 452 × 2 = 904 Total =1356

Hence (C) is correct option.

7. Ans. A.

Given, Cache size = 32 KB = 32 × 2^{10} B; Block size = 32 B = 2^5 B

We know that;

$$\text{Number of sets} = \frac{\text{Cache Size}}{\text{Number of Blocks in a set} \times \text{block Size}}$$

ANSWERS

$$= \frac{32 \times 2^{10}}{2 \times 32} = 2^9$$

Number of tag bits = 32-9-5 = 18
Hence, Time for comparing data = 0.6 + 18/10 = 2.4 ns

8. Ans. D.
A miss is not considered a conflict miss if the block is accessed for the first time.
1st round: (2+2) misses
2nd round: (4+4) misses
∴Total = 4 + (8 × 9) = 76 conflict misses

9. Ans. B.
The given instructions can be written as below:
I1: R1 = R2 + R3
I2: R7 = R1 * R3
I3: R4 = R1 - R5
I4: R3 = R2 + R4
I5: R7 = R8 * R9
An anti-dependency, also known as write-after-read (WAR), occurs when an instruction requires a value that is later updated.
S1: There is an anti-dependence between instructions I2 and I5
False, I2 and I5 don't form any write after read situation.
They both write R7
S2: There is an anti-dependence between instructions I2 and I4
True, I2 reads R3 and I4 writes it.
S3: Within an instruction pipeline an anti-dependence always
creates one or more stalls.
Anti-dependency can be removed by renaming variables.
See following example.
1) B = 3
2) A = B + 1
3) B = 7
Renaming of variables could remove the dependency.
1) B = 3
N. B2 = B
2) A = B2 + 1
3) B = 7
So, Option B is true.

10. Ans. B.
Just assume we have two stages in pipeline. They are G & F taking 3 & 5 seconds.
We have two instances of Uf & Ug. It is like having **dual core processor**. We have two pipeline processors.
So we will do i=1 to 5 in one processor & i= 6 to 10 in another processor.
So they will be done in parallel.
So only focus on i=1 to 5.
In ideal pipeline processor CPI=1 *(First instruction takes full time, after it in every cycle one instruction gets completed unless there is any form of hazard)*
So, from 5 instructions, first one will take (5+3) = 8ns

ANSWERS

For the rest 4 instructions in every 5 ns, 1 instruction will be completed. (Since max (5,3) = 5)
5*4= 20ns
So total time taken = **28 ns** (ans)

11. Ans. C.
The data bus is used to transfer data between the I/O device and memory. • When the last word of data in the DMA transfer is transferred, the DMA controller informs the termination of the transfer to the CPU by means of the interrupt line.
Since there is no overhead for DMA transfer, Therefore
Byte transfer takes 1.22 microseconds (same logic as in part C.).
writing this to memory takes 0.1 microsecond
%age of CPU involvement= (0.1)/(0.1+1.22)= 7.5%

12. Ans. A.
Finding the bit is set or not simply we need to perform an "AND" operation with '1' of that bit if result is 0 then bit is not set and if result is not 0 then bit is set. Now if given "pos" is not 0 then we need to left shift 1 by pos (0 x 1 << pos).

13. Ans. B.
Here value of B incremented by 1 only if carry flag is 1, carry is filled using right rotation, so B will store the no. of is in A0. Hence B. is correct option.

14. Ans. D.
Instruction size are given in words. So first instruction will take 2 words i.e 8 bytes(as 32 bit byte addressable, word size will be 32 bit) so on for 2nd instruction 4 byte, for 3rd 4 bytes, 4th 8 bytes.5th 4 bytes. As 1st instruction starts from 1000 and the size is 8 bytes second instruction address will be 1008, likewise 3rd instruction address will be 1012,4th instruction address 1016,5th instruction address 1024 and halt instruction address will be 1028. Word size is 32 bits (4 bytes). Interrupt occurs after execution of HALT instruction NOT **during**, So address of next instruction will be saved on to the stack which is 1028.

15. Ans. D.
Operation $R_1 \leftarrow M$ [3000] executed once on the other hand.
$R_2 \leftarrow M$ [R_3] and M [R_3] ←executed ten times each.
So, total memory reference.
= 2 × 10+1
= 21

16. Ans. C.
Given, Delay at third stage = 2 slots
Total instructions = 10^9
Number of conditional branches = 20% of 10^9
We know that cycle penalty = delay × conditional branches
= 2 x 0.2 x 10^9= 4 x 10^8
Also it is mention as the clock speed is 1 GHz
So total execution time $= \frac{10^9}{10^9} + 4 \times \frac{10^8}{10^9}$ = 1.4 seconds

17. Ans. D.
Without any other program modification I4 can occupy the delay slot.

ANSWERS

18. Ans. B.

Number of clock cycles needed to execute the following sequence of instructions

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
I_0	IF	ID	OF	PO	PO	PO	WO								
I_1		IF	ID	OF			PO	PO	PO	PO	PO	PO	WO		
I_2			IF	ID			OF						PO	WO	
I_3				IF			ID						OF	PO	WO

So ans is 15.

19. Ans. B.

Given: Five stage instruction pipeline Delays for FI, DI, FO, EI and wo are 5, 7, 10, 8, 6 ns resp.

To find: Time needed to execute 12 instruction prog.

Analysis: Since the max time taken by any stage is 10ns and additional 1 ns is required for delay of buffer. Therefore total time for an instruction to pass from one stage to another is 11ns Now instructions are executed as follows:

$$I_1 . I_2 . I_3 I_4 . I_{10} . I_{11} . I_{12} \longrightarrow$$

execution with time

Now when I_4 is in its execution stage we detect the branch and when I_4 is in WO stage we fetch I_9 so time for execution of instructions from I_1 to I_4 is = 11 * 5 + (4 – 1) * 11 = 88 ns and time for execution of instructions from I_9 to I_{12} is

= 11 * 5 + (4 - 1) * 11 = 88 ns

But we have 11ns when fetching I_9. i.e. I_9 requires only 44 ns additional instead of 55 ns because time for fetching I_9 can be overlap with WO of I_4

Hence total time is = 88 + 88 – 11 = 165 ns

20. Ans. A.

	No. of stages	Stall cycle	Stall frequency	Clock Period	Average access time
Old design	5	2	20%	2.2 ns	P
New design	8	5	20%	1 ns	Q

P =[80%(1 clock) + 20%(1$_{\text{(completion)}}$ + 2$_{\text{(stall clock)}}$)] x $T_{c\text{-}p}$

P =(.8 +.6) x 2.2ns = 3.08 ns

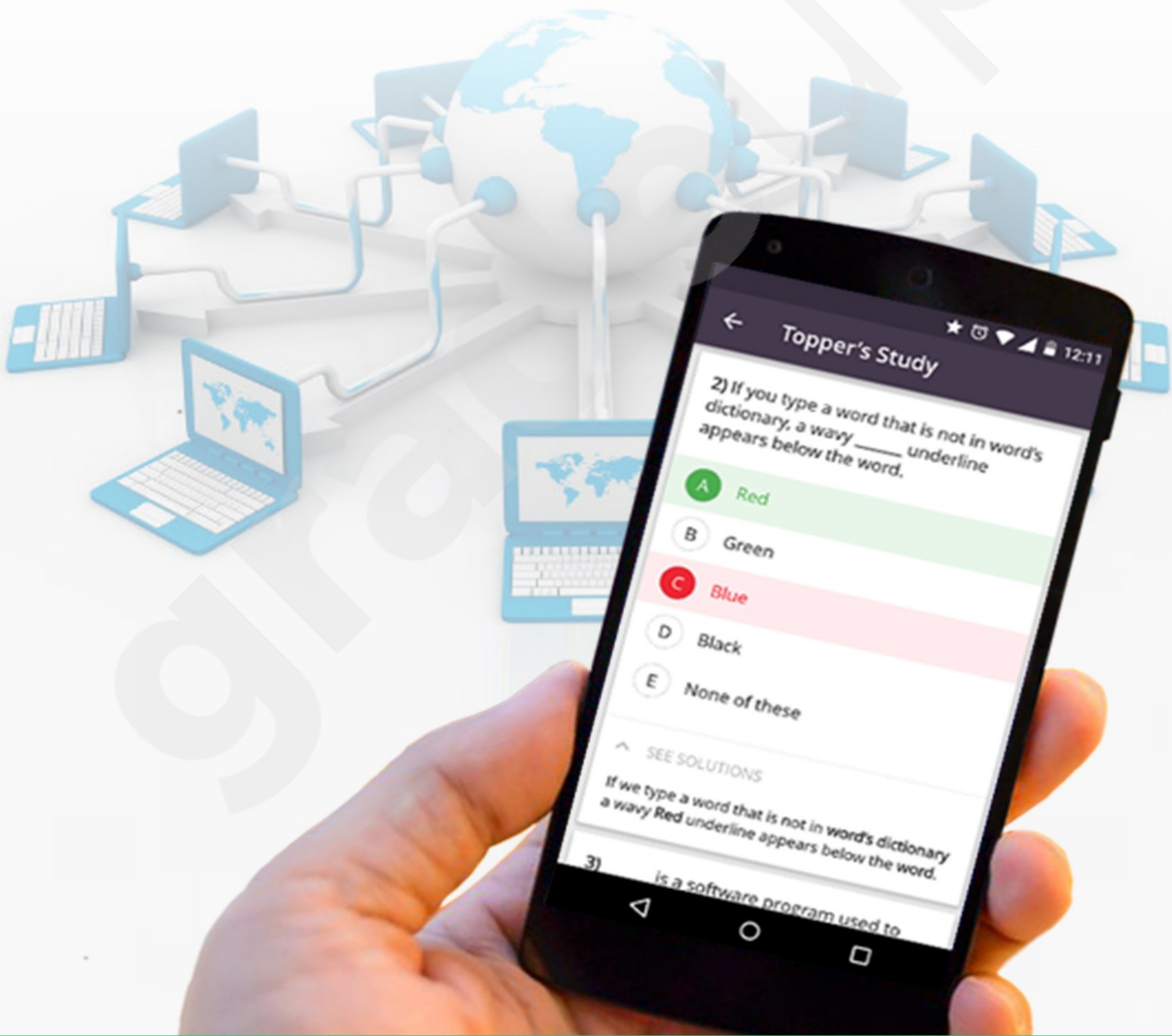
Q =[80%(1 clock) + 20%(1$_{\text{(completion)}}$ + 5$_{\text{(stall clock)}}$)] x $T_{c\text{-}p}$

P =(.8 +.12) x 1 ns = 2 ns

So the value of P/Q = 3.08 ns/ 2 ns = 1.54

gradeup

Computer Network
Questions & Solutions



Computer Networks

1. Host A is sending data to host B over a full duplex link. A and B are using the sliding window protocol for flow control. The send and receive window sizes are 5 packets each. Data packets (sent only from A to B) are all 1000 bytes long and the transmission time for such a packet is 50 μs Acknowledgement packets (sent only from B to A) are very small and require negligibl transmission time. The propagation delay over the link is 200 μs. What is the maximum achievable throughput in this communication?
A. 7.69×10^6 Bytes per second
B. 11.11×10^6 Bytes per second
C. 12.33×10^6 Bytes per second
D. 15.00×10^6 Bytes per second

2. Suppose that the sliding window protocol is used with the sender window size of i, where L is the number of bits identified in the earlier part and acknowledgements are always piggy backed. After sending i frames, what is the minimum time the sender will have to wait before starting transmission of the next frame? (Identify the closest choice ignoring the frame processing time.)
A. 16ms B. 18ms C. 20ms D. 22ms

Direction (3-4): Frames of 1000 bits arc sent over a 10^6 bps duplex link between two hosts. The propagation time is 25ms. Frames are to be transmitted into this link to maximally pack them in transit (within the link).

3. What is the minimum number of bits (l) that will be required to represent the sequence numbers distinct!)? Assume that no time gap needs to be given between transmission of two frames.
A. $l = 2$ B. $l = 3$ C. $l = 4$ D. $l = 5$

4. Consider a network connecting two systems located 8000 kilometers apart. The bandwidth of the network is 500×10^6 bits per second. The propagation speed of the media is 4×10^6meters per second. It is needed to design a Go-Back-N sliding window protocol for this network. The average packet size is 10^7 bits. The network is to be used to its full capacity. Assume that processing delays at nodes are negligible. Then, the minimum size in bits of the sequence number field has to be ____________.
A. 4 B. 6 C. 8 D. 12

5. Suppose that the stop-and-wait protocol is used on a link with a bit rate of 64 kilobits per second and 20 milliseconds propagation delay. Assume that the transmission time for the acknowledgement and the processing time at nodes are negligible. Then the minimum frame size in bytes to achieve a link utilization of at least 50% is _________.
A. 160 B. 320 C. 640 D. 220

6. The values of parameters for the Stop-and – Wait ARQ protocol are as given below:
Bit rate of the transmission channel = 1 Mbps
Propagation delay from sender to receiver = 0.75 ms
Time to process a frame = 0.25 ms
Number of bytes in the information frame = 1980
Number of bytes in the acknowledge frame = 20
Number of overhead bytes in the information frame = 20
Assume that there are no transmission errors. Then the transmission efficiency (expressed in percentage) of the Stop-and – Wait ARQ protocol for the above parameters is _____.
A. 75.33 B. 89.33 C. 92.33 D. 99.33

7. While opening a TCP connection, the initial sequence number is to be derived using a time-of-day (ToD) clock that keeps running even when the host is down. The low order 32 bits of the counter of the ToD clock is to be used for the initial sequence numbers. The clock counters increments once per millisecond. The maximum packet lifetime is given to be 64s.
Which one of the choices given below is closest to the minimum permissible rate at which sequence numbers used for packets of a connection can increase?
A. 0.015/s B. 0.064/s C. 0.135/s D. 0.327/s

Computer Networks

8. Assume that the bandwidth for a TCP connection is 1048560 bits /sec. Let α be the value of RTT in milliseconds (rounded off to the nearest integer) after which the TCP window scale option is needed. Let β be the maximum possible window size with window scale option. Then the values of α and β are

A. 63 milliseconds, 65535 x 2^{14}
B. 63 milliseconds, 65535 x 2^{16}
C. 500 milliseconds, 65535 x 2^{14}
D. 500 milliseconds, 65535 x 2^{16}

9. Consider a TCP client and a TCP server running on two different machines. After completing data transfer, the TCP client calls close to terminate the connectional and a FIN segment is sent to the TCP server. Server-side TCP responds by sending an ACK which is received by the client-side TCP. As per the TCP connections state diagram (RFC 793), in which state does the client-side TCP connection wait for the FIN from the sever-side TCP?

A. LAST-ACK
B. TIME-WAIT
C. FIN-WAIT-1
D. FIN-WAIT-2

10. Suppose two hosts use a TCP connection to transfer a large file. Which of the following statements is/are FALSE with respect to the TCP connection?

I. If the sequence number of a segment is *m*, then the sequencenumber of the subsequent segment is always *m*+1.

II. If the estimated round trip time at any given point of time is *t*sec, the value of the retransmission timeout is always set to greater than or equal to *t*sec.

III. The size of the advertised window never changes during the course of the TCP connection.

IV. The numberof unacknowledged bytesat the sender is alwaysless than or equal to the advertised window.

A. III only
B. I and III only
C. I and IV only
D. II and IV only

11. Consider the following routing table at an IP router:

Network No.	Net Mask	Next Hop
128.96.170.0	255.255.254.0	Interface 0
128.96.168.0	255.255.254.0	Interface 1
128.96.166.0	255.255.254.0	R2
128.96.164.0	255.255.252.0	R3
0.0.0.0	Default	R4

For each IP address in Group I identifythe correct choice of the next hop from Group II using the entries from the routing table above.

Group I

i) 128.96.171.92
ii) 128.96.167.151
iii) 128.96.163.151
iv) 128.96.165.121

Group II

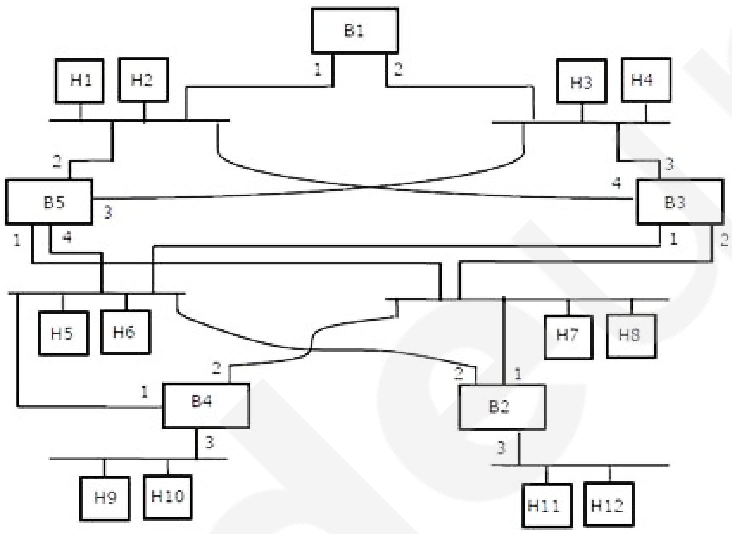
a) Interface 0
b) Interface 1
c) R2
d) R3
e) R4

A. i-a, ii-c, iii-e, iv-d
B. i-a, ii-d, iii-b, iv-e
C. i-b, ii-c, iii-d, iv-e
D. i-b, ii-c, iii-e, iv-d

Computer Networks

12. Consider the diagram shown below where a number of LANs are connected by (transparent) bridges. In order to avoid packets looping through circuits in the graph, the bridges organize themselves in a spanning tree. First, the root bridge is identified as the bridge with the least serial number. Next, the root sends out (one or more) data units to enable the setting up of the spanning tree of shortest paths from the root bridge to each bridge.

Each bridge identifies a port (the root port) through which it will forward frames to the root bridge. Port conflicts are always resolved in favour of the port with the lower index value. When there is a possibility of multiple bridges forwarding to the same LAN (but not through the root port), ties are broken as follows: bridges closest to the root get preference and between such bridges, the one with the lowest serial number is preferred.



For the given connection of LANs by bridges, which one of the following choices represents the depth first traversal of the spanning tree of bridges?

A. B1, B5, B3, B4, B2
B. B1, B3, B5, B2, B4
C. B1, B5, B2, B3, B4
D. B1, B3, B4, B5, B2

13. Consider a source computer (S) transmitting a file of size 10^6 bits to a destination computer (D) over a network of two routers (R_1 and R_2) and three links (L_1, L_2, and L_3). L_1connects S to R_1; L_2 connects R_1 to R_2; and L_3 connects R2 to D. Let each link be of length 100 km. Assume signals travel over each link at a speed of 10^8 meters per second. Assume that the link bandwidth on each link is 1Mbps. Let the file be broken down into 1000 packets each of size 1000 bits. Find the total sum of transmission and propagation delays in transmitting the file from S to D?

A. 1005 ms
B. 1010 ms
C. 3000 ms
D. 3003 ms

14. Let the size of congestion window of a TCP connection be 32 KB when a timeout occurs. The round trip time of the connection is 100 m sec and the maximum segment size used is 2 KB. The time taken (**in m sec**) by the TCP connection to get back to 32 KB congestion window is _________.

A. 1100 to 1300
B. 800 to 1000
C. 1400 to 1600
D. 1500 to 1700

15. A computer network uses polynomials over GF(2) for error checking with 8 bits as information bits and uses $x^3 + x + 1$ as the generator polynomial to generate the check bits. In this network, the message 01011011 is transmitted as

A. 01011011010
B. 01011011011
C. 01011011101
D. 01011011100

16. A and B are the only two stations on an Ethernet. Each has a steady queue of frames to send. Both A and B attempt to transmit a frame, collide, and A wins the first back off race. At the end of this successful transmission by A, both A and B attempt to transmit and collide. The probability that A wins the second back off race is

A. 0.5
B. 0.625
C. 0.75
D. 1.0

Computer Networks

17. A message is made up entirely of characters from the set X= {P, Q, R, S, T}. The table of probabilities for each of the characters is shown below:

Character	Probability
P	0.22
Q	0.34
R	0.17
S	0.19
T	0.08
Total	1.00

If a message of 100 characters over X is encoded using Huffman coding, then the expected length of the encoded message in bits is_____.

A. 225 B. 232 C. 245 D. 255

18. Host A (on TCP/IP v4 network A) sends an IP datagram D to host B (also on TCP/IP v4 network B). Assume that no error occurred during the transmission of D. When D reaches B, which of the following IP header field(s) may be different from that of the original datagram D?

(i) TTL (ii) Checksum (iii) Fragment Offset

A. (i) only B. (i) and (iii) only

C. (ii) and (iii) only D. (i), (ii) and (iii)

19. An IP router with a Maximum Transmission Unit (MTU) of 1500 bytes has received an IP packet of size 4404 bytes with an IP header of length 20 bytes. The values of the relevant fields in the header of the third IP fragment generated by the router for this packet are

A. MF bit: 0, Datagram Length: 1444; Offset: 370

B. MF bit: 1, Datagram Length: 1424; Offset: 185

C. MF bit: 1, Datagram Length: 1500; Offset: 370

D. MF bit: 0, Datagram Length: 1424; Offset: 2960

20. What is the rate at which application data is transferred to host H_C? Ignore errors, acknowledgements, and other overheads.

A. 325.5 Kbps B. 354.5 Kbps C. 409.6 Kbps D. 512.0 Kbps

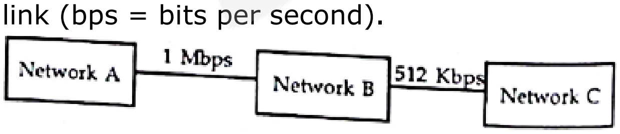
21. Consider three IP networks A, B and C. Host HA in networks A sends messages each containing 180 bytes of application data to a host HC in network C. The TCP layer prefixes a 20 byte header to the message. This passes through an intermediate network B. the maximum packet size, including 20 byte IP header, in each network is:

A : 1000 bytes

B : 100 bytes

C : 1000 bytes

The network A and B are connected through a 1 Mbps link, while B and C are connected by a 512 Kbps link (bps = bits per second).



Assuming that the packets are correctly delivered, how many bytes, including headers, are delivered to the IP layer at the destination for one application message, in the best case? Consider only data packets.

A. 200 B. 220 C. 240 D. 260

ANSWERS

1. Ans. B.
Transmission time =50 micro sec
Propagation time =200 micro sec
RTT= 50+2*200=450 microsec
Total number of bits transmitted before first ACK is received = 1000x5x8 bits =40000 bits
After first ACK is received, same cycle of action repeats.
So, Throughput = (40000/450) x10^6 bits
= 88.88x10^6 bits per second = 11.11 x10^6 Bytes per second

2. Ans. B.
Size of sliding window = 2^5 = 32
Transmission time for a frame = 1ms
Total time taken for 32 frames = 32ms
The sender cannot receive acknowledgement before round trip time which is 50ms.
After sending 32 frames, the minimum time the sender will have to wait before starting transmission of the next frame = 50 – 32 = 18

3. Ans. D.
$$Pulse\ rate = \frac{number\ of\ bits}{time\ taken} = \frac{1000}{25\times10^{-3}} = 4\times10^{4}$$
Alternately
Bit rate = pulse rate × 2' 10^3 ms …… 10 bits
Here, 2' = 25 $50" .... = \frac{1000}{50} = 20\,bits$
$2^4 < 2^5 < 2'$
So *l* = 5 Minimum number of bits required is 5.

4. Ans. C.
Propagation time = (8000 * 1000)/ (4 * 10^6) = 2 seconds
Total round trip propagation time = 4 seconds
Transmission time for one packet = (packet size) / (bandwidth)
= (10^7) / (500 * 10^6) = 0.02 seconds
Total number of packets that can be transferred before an
Acknowledgement comes back = 4 / 0.02 = 200
Maximum possible window size is 200.
In Go-Back-N, maximum sequence number should be one more than
Window size.
So total 201 sequence numbers are needed. 201 different sequences
Numbers can be represented using 8 bits. the minimum size in bits of the sequence number field has to be 4.

5. Ans. B.
Transmission or Link speed = 64 kb per sec
Propagation Delay = 20 milisec
Since stop and wait is used, a packet is sent only
when previous one is acknowledged.
Let x be size of packet, transmission time = x / 64 milisec

ANSWERS

Since utilization is at least 50%, minimum possible total time for one packet is twice of transmission delay, which means

x/64 * 2 = x/32

x/32 > x/64 + 2*20

x/64 > 40

x > 2560 bits = 320 bytes

6. Ans. B.

Given Data:

$B = 1 Mbps$

$T_{proc} = 0.25\,ms$

$T_p = 0.75\,ms$

$L = 1980\,Bytes$

$L_{OH} = 20\ Bytes$

$L_A = 20\ Bytes$

$Efficiency(\eta) = ?$

(i) $T_x = \frac{L}{B} = \frac{(1980+20)\times 8}{10^6} = \frac{2\times 8\times 10^3}{10^6}$

= 16 ms

(ii) $T_{ACK} = \frac{L_A}{B} = \frac{20\times 8}{10^6} = 0.16\ ms$

In stop-and-wait ARQ, efficiency

$$\eta = \frac{T_x}{T_x + T_{AcK} + 2T_p + T_{proc}} = \frac{16\,ms}{17.91\,ms}$$

$$= 0.8933 \approx 89.33\%$$

7. Ans. A.

The maximum packet lifetime is given to be 64 seconds in the question.

Thus, a sequence number increments after every 64 seconds.

So, minimum permissible rate = 1 / 64 = 0.015 per second

8. Ans. C.

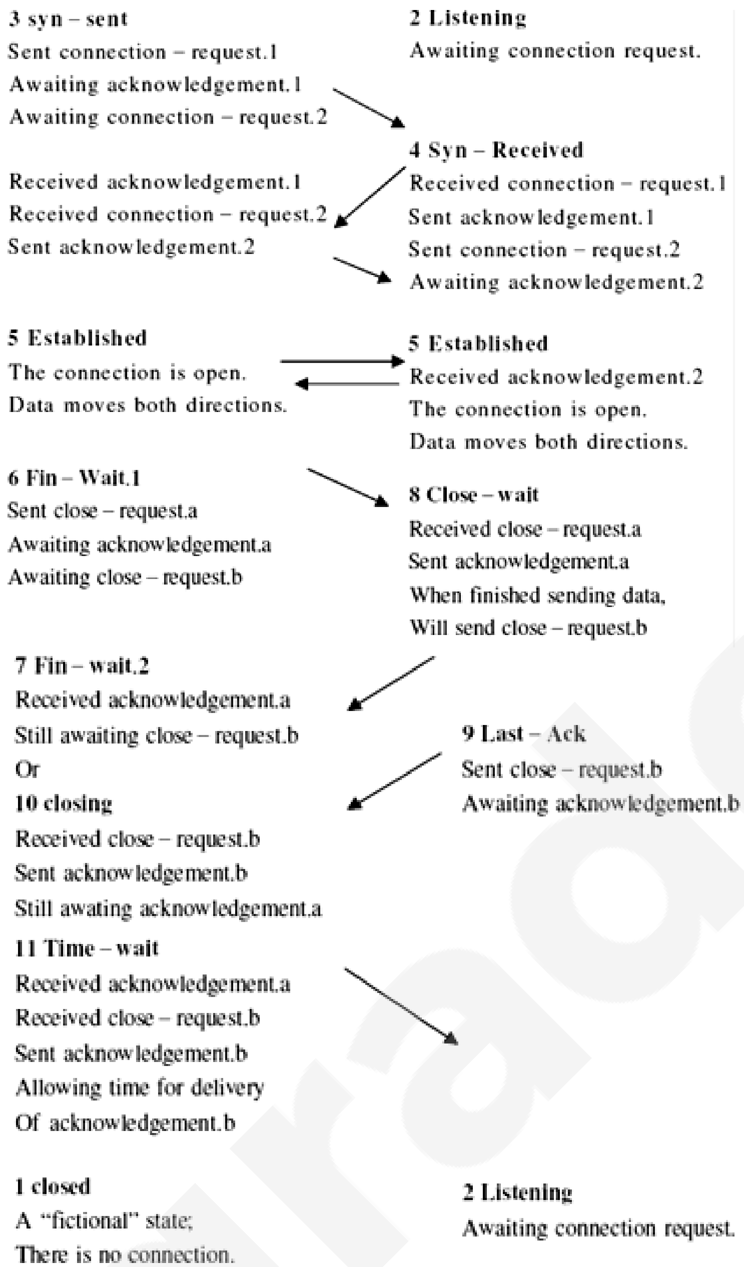
Since sequence number in TCP header is limited to 16 bits, the maximum window size is limited. When bandwidth delay product of a link is high, scaling is required to efficiently use link. TCP allows scaling of windows when bandwidth delay product is greater than 65,535 (Refer this). The bandwidth delay product for given link is 1048560 * α. Window scaling is needed when this value is more than 65535 bytes, i.e., when α is greater than 65535 * 8 / 1048560 or 0.5 seconds. Scaling is done by specifying a one byte shift count in the header options field. The true receive window size is left shifted by the value in shift count. A maximum value of 14 may be used for the shift count value. Therefore maximum window size with scaling option is 65535×2^{14}.

9. Ans. D.

Client* Server*

*or vice-versa, though requests typically originate at clients.

ANSWERS



10. Ans. B.

TCP sequence number of a segment is the byte number of the first byte in the segment. For example, if the segment contains 500 bytes which are from 1000 to 1499, then sequence number of the segment would be 1000 and sequence number of next segment would be 1500. Receiver window changes when TCP data is processed by application layer of receiver side.

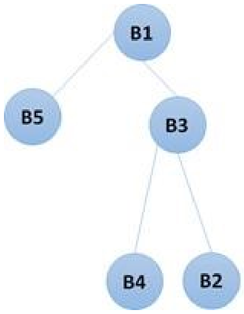
11. Ans. A.

The next hop is decided according to the longest prefix matching. Next hop is a routing term that refers to the next closest router a packet can go through. The next hop is among the series of routers that are connected together in a network and is the next possible destination for a data packet. The Internet consists of thousands of different networks of every size and shape. Routers are among the most important and significant network devices in this network in that they hold the key to the rapid growth of the Internet worldwide, enabling communication among the devices. Therefore, a router has to manage the information related to its topological surroundings, specifically about nearby routers. Whenever a router maintains information about the routers in its routing table, the lowest metric among them is known as the next hop or the next optimal router.

ANSWERS

12. Ans. A.

As we need shortest path from the Source Root to any other bridge, hence the spanning tree will be,



13. Ans. A.

Transmission delay for 1 packet from each of S, R_1 and R_2 will take 1 ms
Propagation delay on each link L_1, L_2 and L_3 for one packet is 1ms
Therefore the sum of transmission delay and propagation delay on each link for one packet is 2ms.
The first packet reaches the destination at 6^{th} ms
The second packet reaches the destination at 7^{th} ms
So inductively we can say that 1000^{th} packet reaches the destination at 1005^{th} ms

14. Ans. A.

Here, Congestion Window Size (CWS) is 32KB and Round Trip Time (RTT) = l00ms. When Time Out occurs, for the next round of Slow Start,
Threshold = (size of CWS) /2 i.e. Threshold = 16KB
Slow Start
2KB
I RTT
4KB
2 RTT
8KB
3RTT
16KB -------------->Threshold reaches. So Additive Increase Starts
4 RTT
18KB
5RTT
20K B
6RTT
22KB
7RTT
24KB
8RTT
26KB
9RTT
28KB
I0RTT
30KB
11RTT
32KB
So, Total no. of RTT's = 11 → 11 * 100 = 1100

ANSWERS

15. Ans. C.

```
Message : 0 1 0 1 1 0 1 1
GF:   x^3 + x + 1 = 1 0 1 1


                0 1 0 0 0 0 1 1
            --------------------------
1 0  1 1 / 0 1 0 1 1 0 1 1 0 0 0
           0 0 0 0 | | | | | | |
           --------- | | | | | |
             1 0 1 1 | | | | | |
             1 0 1 1 | | | | | |
             --------- | | | | |
               0 0 0 0 1 1 0 0 |
                       1 0 1 1 |
                       ---------
                         1 1 1 0
                         1 0 1 1
                         -------|
                           1 0 1 (remainder )
```

Answer: Message transmitted :0 1 0 1 1 0 1 1 1 0 1 (OPTION C)

16. Ans. B.

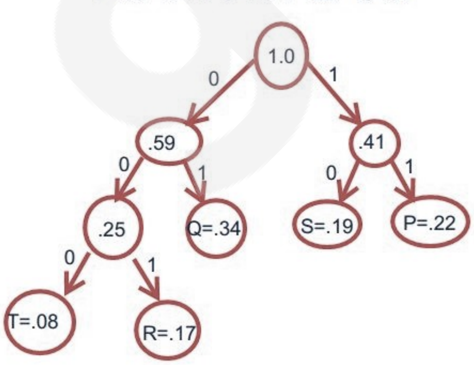
At every attempt to transit a frame, both A and B chooses value of ‘k’ randomly. Based on the value of ‘k’, back-off time is calculated as a multiple of ‘k’. The station or node having the smaller back-off time gets to send the frames earlier.

1st attempt: Value of ‘k’ would be k=0 or k=1 (0 <= k <= 2^n-1; where n=nth attempt). Since A won the first race, A must have chosen k=0 and B must have chosen k=1 (A wins here with probability 0.25). As A won, A will again choose k=0 or k=1 for its 2nd frame, but B will choose k=0, 1, 2 or 3 as B failed to send its first frame in the first attempt. 2nd attempt: Let kA= value of k chosen by A and kB = value of k chosen by B. We will use notation (kA, kB) to show the possible values. Now the sample space for the 2nd attempt is (kA, kB) = (0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2) or (1,3) i.e. 8 possible outcomes. For A to win, kA should be less than kB (kA < kB). Thus, our event space is (kA, kB) = (0,1),(0,2),(0,3),(1,2),(1,3) i.e. 5 possible outcomes. Thus the probability that A wins the 2nd back-off race = 5/8 = 0.625

17. Ans. A.

Huffman tree is as follows

T=.08 R=.17 S=.19 P=.22 Q=.34



no of bit required for each alphbet:
T=3bit, R=3bit, Q=2bit, S=2bit, P=2bit
agv length/char=no of bit* frequency of occurance of each alphabet
=3*.08+3*.17+2*.34+2*.19+2*.22=2.25bits
so for 100 char=2.25*100=225bits

ANSWERS

18. Ans. B.
All (i), (ii) and (iii) are changed.
TTL is decremented at every hop. So TTL is different from original value.
Since TTL changes, the Checksum of the packet also changes.
A packet is fragmented if it has a size greater than the maximum transmission Unit (MTU) of the network. There may be intermediate networks that may change fragment offset by fragmenting the packet.

19. Ans. A.
Number of packet fragments = ⌈ (total size of packet)/(MTU) ⌉
= ⌈4404/1500 ⌉
= ⌈2.936 ⌉
= 3
So Datagram with data 4404 byte fragmented into 3 fragments. The first frame carries bytes 0 to 1479 (because MTU is 1500 bytes and HLEN is 20 byte so the total bytes in fragments is maximum 1500-20=1478). The offset for this datagram is 0/8 = 0. The second fragment carries byte 1480 to 2959. The offset for this datagram is 1480/8 = 185. Finally the third fragment carries byte 2960 to 4404. The offset is 370 and for all fragments except last one the M bit is 1. So in the third bit M is 0.

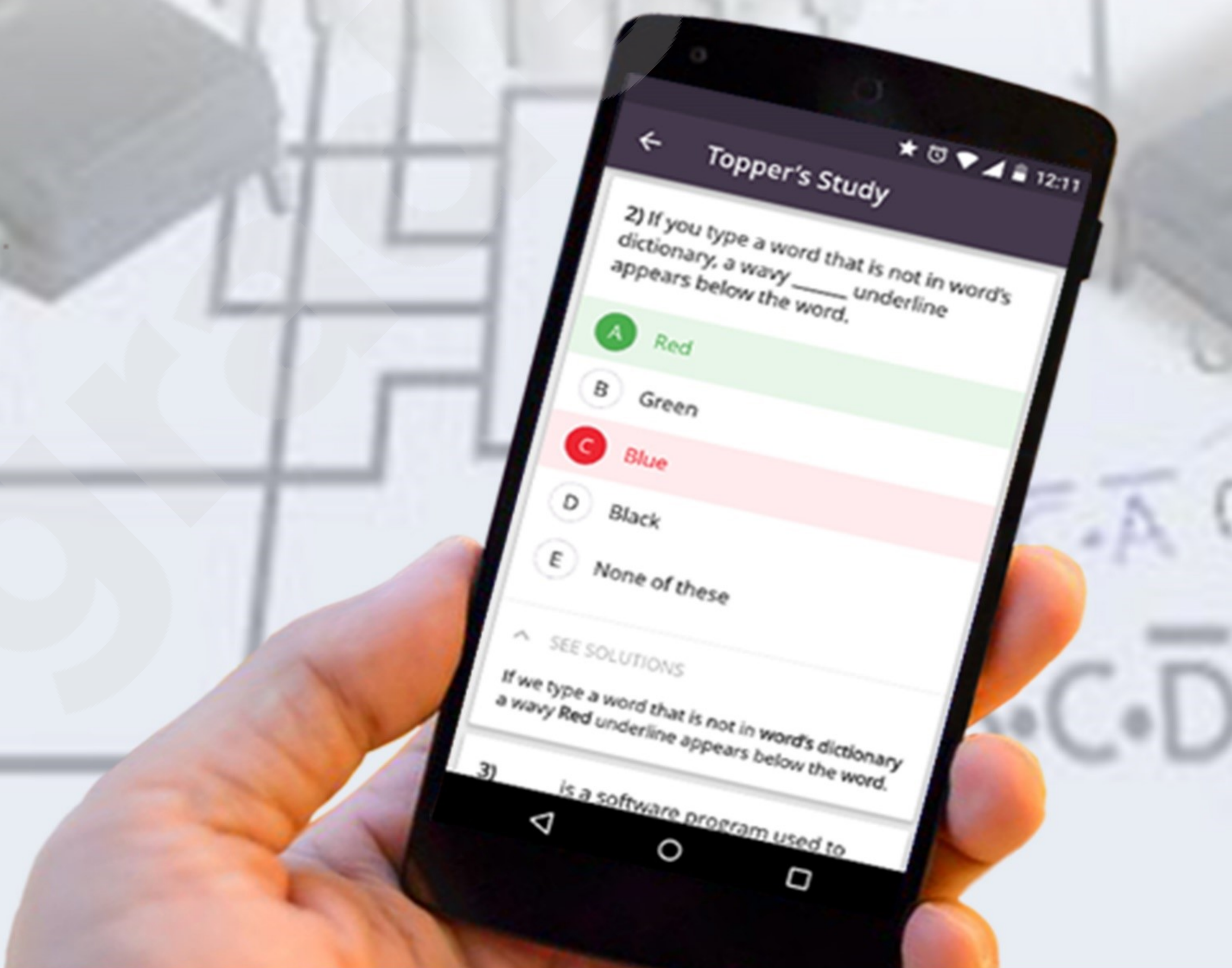
20. Ans. B.
Time to transfer 260 bytes from B-C = 260 * 8/(512 * 1000) = 65/16000 = 13/3200
So, data rate = 180 * 3200 / 13 = 44.3 kBps = 354.46 kbps = 354.5 Kbps

21. Ans. D.

As 3 packets are formed
Packet A sends an IP packet of 180 bytes of data + 20 bytes of TCP header + 20 bytes of IP header to B.
IP layer of B now removes 20 bytes of IP header and has 200 bytes of data.
So, it makes 3 IP packets - [80 + 20, 80 + 20 + 40 + 20] and sends to C as the Ip packet size of B is 100.
C receives 260 bytes of data which includes 60 bytes of IP headers and 20 bytes of TCP header.

gradeup

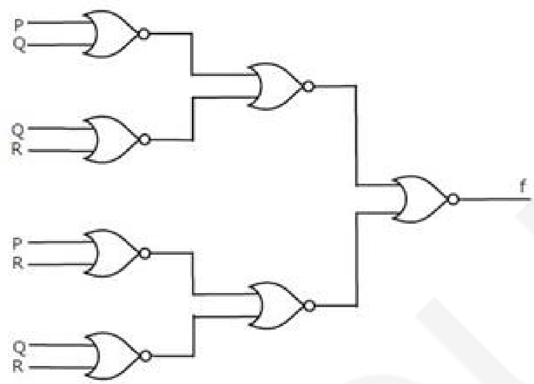
Digital Logic
Questions & Solutions



Digital Logic

1. The number of min-terms after minimizing the following Boolean expression is_________.
[D′ + AB′ + A′C + AC′D + A′C′D]′
A. 1 B. 46 C. 56 D. 76

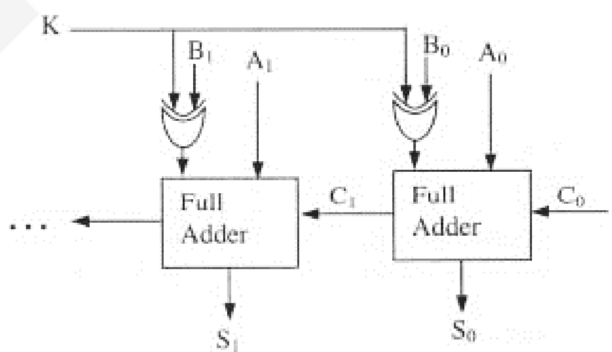
2. What is the Boolean expression for the output f of the combinational logic circuit of NOR gates given below?



A. $\overline{Q+R}$ B. $\overline{P+Q}$ C. $\overline{P+R}$ D. $\overline{P+Q+R}$

3. Given the function F = P′ + QR, where F is a function in three Boolean variables P, Q and R and P′ =! P, consider the following statements.
(S1) F=$^{\Sigma}$ (4, 5, 6)
(S2)F=$^{\Sigma}$ (0, 1, 2, 3, 7)
(S3)F=$^{\Pi}$ (4, 5, 6)
(S4)F=$^{\Pi}$ (0, 1, 2, 3, 7)
Which of the following is true?
A. (S1)-False, (S2)-True, (S3)-True, (S4)-False
B. (S1)-True, (S2)-False, (S3)-False, (S4)-True
C. (S1)-False, (S2)-False, (S3)-True, (S4)-True
D. (S1)-True, (S2)-True, (S3)-False, (S4)- False

4. Consider the ALU shown below.



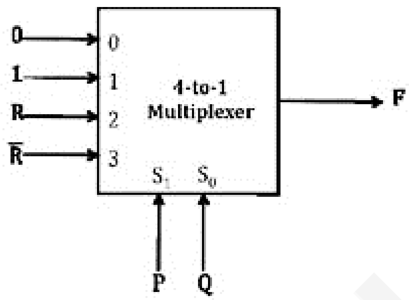
If the operands are in 2's complement representation, which of the following operations can be perforModerate by suitably setting the control lines K and C0 only (+ and – denote addition and subtraction respectively)?
A. A + B, and A - B, but not A + 1 B. A + B, and A + 1, but not A – B
C. A + B, but not A – B or A + 1 D. A + B, and A - B, and A + 1

Digital Logic

5. A half adder is implemented with XOR and AND gates. A full adder is implemented with two half adders and one OR gate. The propagation delay of an XOR gate is twice that of an AND/OR gate. The propagation delay of an AND/OR gate is 1.2 microseconds. A 4-bit ripple-carry binary adder is implemented by using four full adders. The total propagation time of this 4-bit binary adder in microseconds is____________.

A. 19.2 ms B. 20 ms C. 15 ms D. 45 ms

6. Consider the 4-to-1 multiplexer with two select lines S_1 and S_0 given below.



The minimal sum-of-products form of the Boolean expression for the output F of the multiplexer is

A. $\bar{P}Q + Q\bar{R} + P\bar{Q}R$

B. $\bar{P}Q + \bar{P}Q\bar{R} + PQ\bar{R} + P\bar{Q}R$

C. $\bar{P}QR + \bar{P}Q\bar{R} + Q\bar{R} + P\bar{Q}R$

D. $PQ\bar{R}$

7. When two 8-bit numbers $A_7 \ldots A_0$ and $B_7 \ldots B_0$ in 2's complement representation (with A_0 and B_0 as the least significant bits) are added using a ripple-carry adder, the sum bits obtained are $S_7 \ldots S_0$ and the carry bits are $C_7 \ldots C_0$. An overflow is said to have occurred if

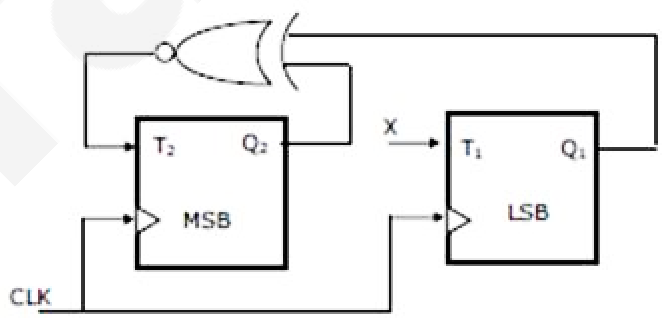
A. the carry bit C_7 is 1

B. all the carry bits $C_7 \ldots C_0$ are 1

C. $\left(A_7.B_7.\overline{S_7} + \overline{A_7}.\overline{B_7}.S_7\right)$ is 1

D. $\left(A_0.B_0.\overline{S_0} + \overline{A_0}.\overline{B_0}.S_0\right)$ is 1

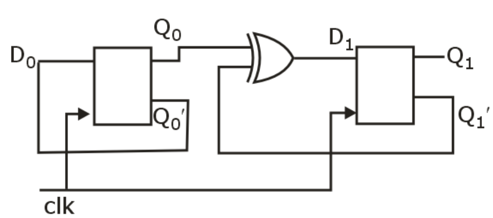
8. Consider the partial implementation of a 2-bit counter using T flip-flops following the sequence 0-2-3-1-0, as shown below.



To complete the circuit, the input X should be

A. Q_2' B. $Q_2 + Q_1$ C. $(Q_1 \oplus Q_2)'$ D. $Q_1 \oplus Q_2$

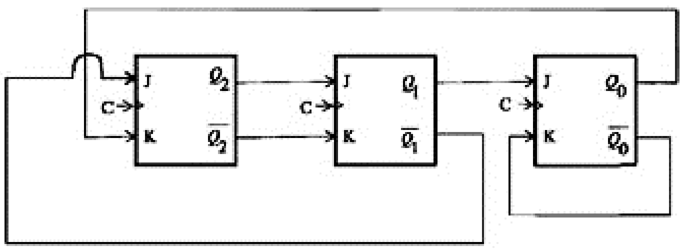
9. Consider the following circuit



Digital Logic

The flip-flops are positive edge triggered D FFs. Each state is designated as a two bit string Q_0Q_1. Let the initial state be 00. The state transition sequence is:

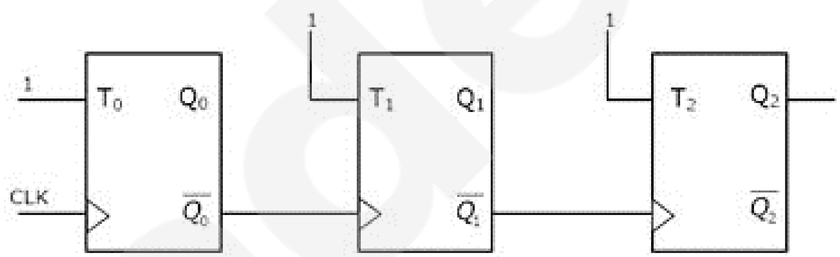
A. $00 \rightarrow 11 \rightarrow 01$ (returning to 00)
B. $00 \rightarrow 11$ (returning to 00)
C. $00 \rightarrow 10 \rightarrow 01 \rightarrow 11$ (returning to 00)
D. $00 \rightarrow 11 \rightarrow 01 \rightarrow 10$ (returning to 00)

10. 

The above synchronous sequential circuit built using JK flip-flops is initialized with Q2Q1Q0 = 000. The state sequence for this circuit for the next 3 clock cycles is

A. 001, 010, 011
B. 111, 110, 101
C. 100, 110, 111
D. 100, 011, 001

11. Figure shows a ripple counter using positive edge triggered flip-flops. If the present state of counter is $_2$. $_1$. $_0$. = 011 then its next state ($_2$. $_1$. $_0$.) will be



A. 010
B. 111
C. 100
D. 101

12. We want to design a synchronous counter that counts the sequence 0-1-0-2-0-3 and then repeats. The minimum number of J-K flip-flops required to implement this counter is _________.

A. 2
B. 3
C. 4
D. 5

13. We consider the addition of two 2's complement numbers $b^{n-1}b^{n-2}$......b_0 and a_{n-1} a_{n-2}.....a_0. A binary adder for adding unsigned binary numbers is used to add the two numbers. The sum is denoted by $c_{n-1}c_{n-2}$.......$c_0$and the carry-out by c_{out}. Which one of the following options correctly identifies the overflow condition?

A. $c_{out}\left(\overline{a_{n-1} \oplus b_{n-1}}\right)$
B. $a_{n-1}b_{n-1}\overline{c_{n-1}} + \overline{a_{n-1}b_{n-1}}\,c_{n-1}$
C. $c_{out} \oplus c_{n-1}$
D. $a_{n-1} \oplus b_{n-1} \oplus c_{n-1}$

ANSWERS

1. Ans. A.
Given Boolean expression is:
[D′ + AB′ + A′C + AC′D + A′C′D]′
Step 1: [D′ + AB′ + A′C + C′D (A + A')]′
(taking C'D as common)
Step 2: [D′ + AB′ + A′C + C′D]′
(as, A + A' = 1)
: [D' + DC' + AB' + A'C]' (Rearrange)
Step 3: [D' + C' + AB' + A'C]'
(Rule of Duality, A + A'B = A + B)
: [D' + C' + CA' + AB']' (Rearrange)
Step 4: [D' + C' + A' + AB']'
(Rule of Duality)
: [D' + C' + A' + AB']' (Rearrange)
Step 5: [D' + C' + A' + B']'
(Rule of Duality)
:[(D' + C')'.(A' + B')']
(Demorgan's law, (A + B)'=(A'. B'))
:[(D''.C'').(A''.B'')] (Demorgan's law)
:[(D.C).(A.B)] (Idempotent law, A'' = A)
: ABCD
Hence only 1 minterm after minimization.

2. Ans. A.
Output is

$$\overline{\left(\overline{(P+Q)}+\overline{(Q+R)}\right)}+\overline{\left(\overline{(P+R)}+\overline{(Q+R)}\right)}$$

$$\left(\overline{P+Q}+\overline{Q+R}\right).\left(\overline{(P+R)}+\overline{Q+R}\right)\left(\because\left(\overline{A+B}\right)=\bar{A}\bar{B}\right)$$

$$\Rightarrow\left(\bar{P}\bar{Q}+\bar{Q}\bar{R}\right)\left(\bar{P}\,\bar{R}+\bar{Q}\bar{R}\right)$$

$$\Rightarrow\bar{P}\bar{Q}\bar{P}\bar{R}+\bar{P}\bar{O}\bar{Q}\bar{R}+\bar{P}\bar{Q}\bar{R}\bar{R}+\bar{Q}\bar{R}\bar{Q}\bar{R}$$

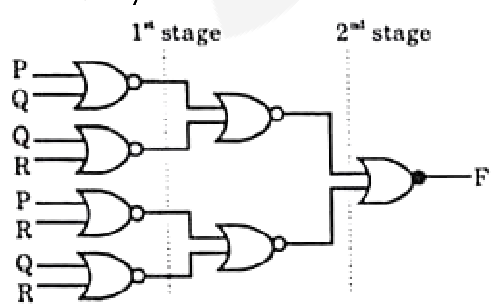
$$\Rightarrow\bar{P}\bar{Q}\bar{R}+\bar{Q}\bar{P}\bar{R}+\bar{P}\bar{Q}\bar{R}+\bar{Q}\bar{R}$$

$$\Rightarrow\bar{P}\bar{Q}\bar{R}+\bar{Q}\bar{R}$$

$\Rightarrow\bar{Q}\bar{R}(\bar{P}+1)$ .....[since $(\bar{P}+1)=1$]

$$\Rightarrow\overline{(Q+R)}$$

Alternately



At first stage

ANSWERS

- $\overline{(P+Q)}$
- $\overline{(Q+R)}$
- $\overline{(P+R)}$
- $\overline{(Q+R)}$

At second stage

- $\overline{\overline{(P+Q)}+\overline{(Q+R)}}$
- $\overline{\overline{(P+R)}+\overline{(Q+R)}}$

$$F=\overline{\overline{\overline{(P+Q)}+\overline{(Q+R)}+\overline{(P+R)}+\overline{(Q+R)}}}$$

$$=\overline{\overline{\overline{(P+Q)}+\overline{(Q+R)}\bullet\overline{(P+R)}+\overline{(Q+R)}}}$$

$$=\left(\overline{(P+Q)}+\overline{(Q+R)}\bullet\overline{(P+R)}+\overline{(Q+R)}\right)$$

$$=(P'Q'+Q'R')\cdot(P'R'+Q'R')$$

$$=\underline{P'Q'R'+P'Q'R'+P'Q'R'+P'Q'R'}+Q'R'$$

$$=P'Q'R'+Q'R'$$

$$=Q'R'(P'+1)$$

$$=Q'R'$$

$$=\overline{Q+R}$$

3. Ans. A.

Option A (S1)-False, (S2)-True, (S3)-True, (S4)-False is correct. After drawing K map of F= P` +QR, we can find out S2 and S3 are TRUE. Karnaugh maps are used to simplify real-world logic requirements so that they can be implemented using a minimum number of physical logic gates. A sum-of-products expression can always be implemented using AND gates feeding into an OR gate, and a product-of-sums expression leads to OR gates feeding an AND gate. Karnaugh maps can also be used to simplify logic expressions in software design. Boolean conditions, as used for example in conditional statements, can get very complicated, which makes the code difficult to read and to maintain. Once minimized, canonical sum-of-products and product-of-sums expressions can be implemented directly using AND and OR logic operators.

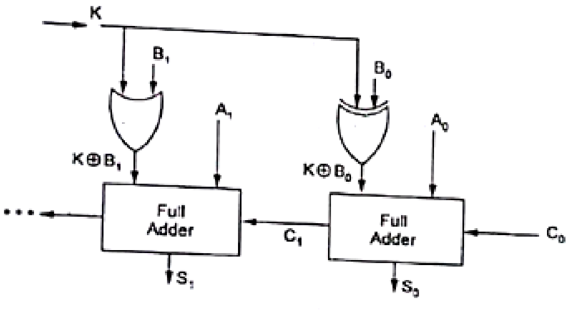
Karnaugh maps are used to facilitate the simplification of Boolean algebra functions. Take the Boolean function described by the following truth table.

Truth table of a function

	A	B	C	D	f(A, B, C, D)
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	0

ANSWERS

4. Ans. A.



If $k = 1, C_0 = 1$

$S_0 = \overline{B_0} \oplus A_0 \oplus C_0 = \overline{B_0} \oplus \overline{A_0}$

$= A_0 \overline{B_0} + B_0 \overline{A_0}$

$C = \overline{B_0} A_0 + (\overline{B_0} \oplus A_0) C_0$

$= \overline{B_0} A_0 + B_0 A_0 + \overline{A_0}\, \overline{B_0}$

$= A_0 + \overline{A_0}\, \overline{B_0}$

$S = A_1 \oplus \overline{B_1} \oplus C_1$

If $k = 0, C_0 = 1$

$S_0 = A_0 \oplus \overline{B_0} \oplus 1 = A_0 \oplus \overline{B_0}$

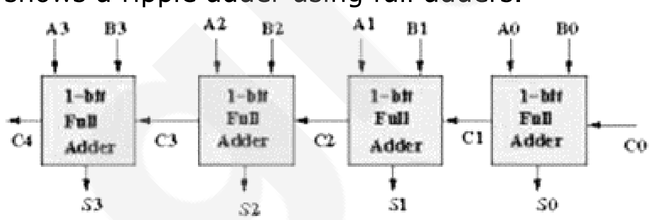
$= A_0 B_0 + \overline{A_0}\, \overline{B_0}$

$C = A_0 B_0 + (A_0 \oplus B_0) = A_0 + \overline{A_0}\, \overline{B_0}$

= A+B and A-B but not A – 1

5. Ans. A.

A Ripple Carry Adder allows adding two n-bit numbers. It uses half and full adders. Following diagram shows a ripple adder using full adders.



Let us first calculate propagation delay of a single 1 bit full adder.

Propagation Delay by n bit full adder is (2n + 2) Gate delays.

[See this for formula].

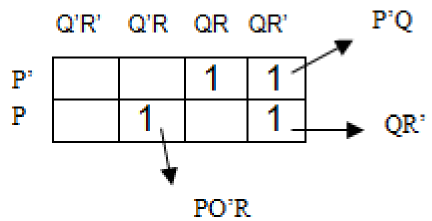
Here n = 1, so total delay of a 1 bit full adder is (2 + 2)*1.2 = 4.8 ms

Delay of 4 full adders is = 4 * 4.8 = 19.2 ms

6. Ans. A.

We got the F= P'Q'(0)+P'Q(1)+PQ'R+PQR' of 4X1 Mux .

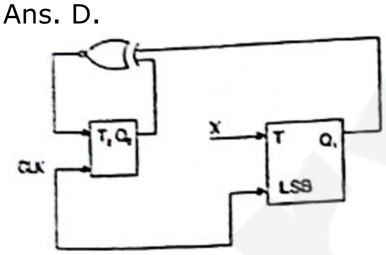
ANSWERS



=P’Q+PQ’R+PQR’
=Q(P’+PR’)+PQ’R
= Q(P’+R’)+PQ’R
=P’Q+QR’+PQ’R

7. Ans. C.
Overflow flag indicates an over flow condition for a signed operation. Some points to remember in a signed operation:
* MSB is always reserved to indicate sign of the number.
* Negative numbers are represented in 2’s – complement.
* An overflow results in invalid operation.
2's complement overflow rules:
* If the sum of two positive numbers yields a negative result, the sum has- overflowed.
* If the sum of two negative number yields a positive result, the sum has overflowed.
* Otherwise, the sum has not overflowed.
Overflow for signed numbers occurs when the carry-in into the MSB (most significant bit) is not equal to carry-out. Conveniently, an XOR-operation on these two bits can quickly determine if an overflow condition exists.
Therefore, $((A_7.B_7)) \oplus S_7 = \overline{A_7}.\overline{B_7}.S_7 + A_7.B_7.S_7 = 1)$ has overflowed.

8. Ans. D.



From circuit, we see T1=XQ1’+X’Q1—-(1)
AND T2=(Q2+Q1)’—-(2)
AND DESIRED OUTPUT IS 00->10->11->01->00
SO X SHOULD BE **Q1Q2’+Q1’Q2** SATISFYING 1 AND 2.
SO ANS IS (D) PART.

9. Ans. D.
Q_0 will toggle in every cycle because Q_0' (Q_0 complement) is fed as input to the D_0flip flop. For the D_1 flip flop, $D_1 = Q_0 \oplus Q_1'$, i.e., Q_0 XOR Q_1'. So, the bit pattern Q_0 Q_1 will be:
Q_0 Q_1
0 0
1 1
0 1
1 0
0 0
Thus, the transition sequence will be $00 \to 11 \to 01 \to 10$ So, D would be the correct choice.

ANSWERS

10. Ans. C.

JK ff truth table

j	k	Q
0	0	Q0
1	0	1
0	1	0
1	1	Q0′

Initially Q2Q1Q0=000 Present state FF input Next state

Q2	Q1	Q0	J2	K2	J1	K1	J0	K0	Q2	Q1	Q0
0	0	0	1	0	0	1	0	1	1	0	0
1	0	0	1	0	1	0	0	1	1	1	0
1	1	0	0	0	1	0	1	1	1	1	1

So answer is C

11. Ans. C.

Since $T_2T_1T_0$ is at 111 i.e., input to these flip flop is always 1. at every clock 2.1.0. will be changes. we have to draw the state diagram for the given circuit. After drawing the whole table,you can clearly see If present state is 011, the next state will be 100.

12. Ans. C.

0->1->0->2->0->3
0000->0001->0100->0010->1000->0011
There are 6 states and 3 of them correspond to same state.
To differentiate between 0,1,2,3 we need 2 bits.
To differentiate between 3 0's we need 2 bits.
So Total 4 bits required =4 FF

13. Ans. B.

Let us draw the truth table for the given condition.

INPUTS			OUTPUTS		
A	B	CARRY IN	CARRY OUT	SUM	OVERFLOW
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	1	0
0	1	1	1	0	0
1	0	0	0	1	0
1	0	1	1	0	0
1	1	0	1	0	1
1	1	1	1	1	0

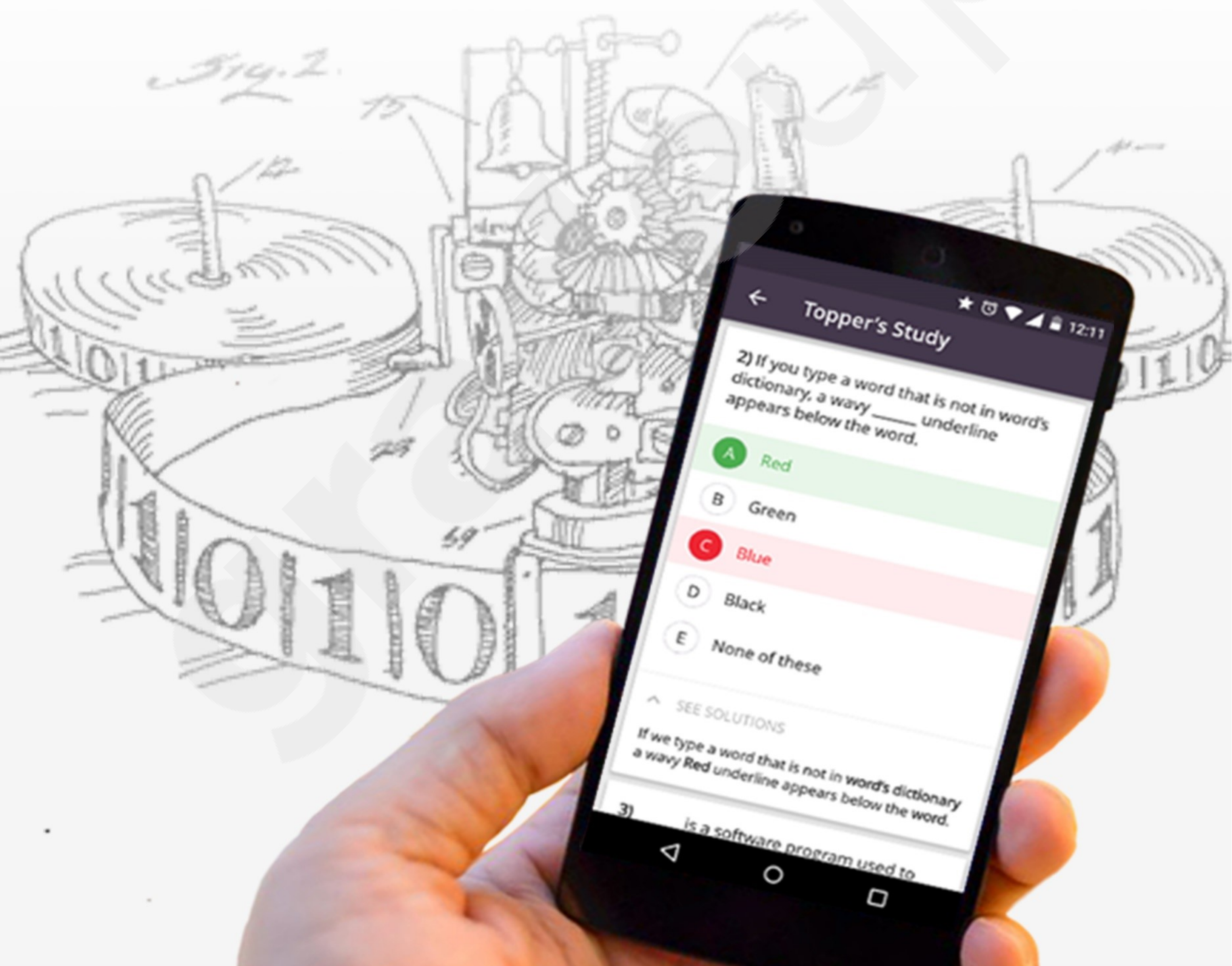
If we look at the above table we find that, value in overflow is 1 only when;
CARRY IN ≠ CARRY OUT

Hence, Overflow= $a_{n-1}b_{n-1}\overline{c_{n-1}} + \overline{a_{n-1}b_{n-1}}\,c_{n-1}$

Or
Counter example for options
(A) 0111+0111=1110 has overflow, but given condition violates.
(C) 1001+0001=1010 has no overflow, but given condition violates.
(D) 1111+1111=1110 has no overflow, but given condition violates.
Only option (B) is correct.

gradeup

Theory of Computation Questions & Solutions



Theory of Computation

1. Consider the following context-free grammar over the alphabet Σ = {a,b,c} with S as the start symbol.

$$S \rightarrow abScT \mid abcT$$

$$T \rightarrow bT \mid b$$

Which one of the following represents the language generated by the above grammar?

A. $\{(ab)^n(cb)^n \mid n \geq 1\}$

B. $\{(ab)^n cb^{m_1}cb^{m_2}...cb^{m_n} \mid n, m_1, m_2, ...m_n \geq 1\}$

C. $\{(ab)^n(cb^m)^n \mid m,n \geq 1\}$

D. $\{(ab)^n(cb^n)^m \mid m,n \geq 1\}$

2. Consider the following languages over the alphabet Σ= {a,b,c}. Let L_1 = {a^n $b^n c^m$ | m, n >= 0 } and L_2 = {$a^m b^n c^n$| m, n >= 0}.

Which of the following are context-free languages?

I. $L_1 \cup L_2$

II. $L_1 \cap L_2$

A. I only B. II only C. I and II D. Neither I nor II

3. Let L_1, L_2 be any two context free languages and R be any regular language. Then which of the following is/are CORRECT?

I. $L_1 \cup L_2$ is context – free

II. $\overline{L_1}$ is context – free

III. $L_1 - R$ is context – free

IV. $L_1 \cap L_2$ is context – free

A. I, II and IV only

B. I and III only

C. II and IV only

D. I only

4. The minimum possible number of states of a deterministic automaton that accepts the regular language L = {w1aw2 | w1, w2 ∊ {a,b}*, |w1|=2,|w2|≥3} is _____.

A. 6 B. 7 C. 8 D. 9

5. Let δ denote the transition function and δˆ denote the extended transition function of the ϵ-NFA whose transition table is given below:

δ	ϵ	a	b
→q0	{q2}	{q1}	{q0}
q1	{q2}	{q2}	{q3}
{q2}	{q0}	∅	∅
q3	∅	∅	{q2}

Then δˆ(q2,aba) is

A. $\emptyset$ B. $\{q_0, q_1, q_3\}$ C. $\{q_0, q_1, q_2\}$ D. $\{q_0, q_2, q_3\}$

Theory of Computation

6. Let L(R) be the language represented by regular expression R. Let L(G) be the language generated by a context free grammar G. Let L (M) be the language accepted by a Turning machine M. Which of the following decision problems are undecidable?

I. Given a regular expression R and a string w, is $w \in L(R)$?

II. Given a context-free grammar G, $L(G) = \varnothing$?

III. Given a context-free grammar G, is $L(G) = \Sigma^*$ for some alphabet Σ?

IV. Given a Turning machine M and a string w, is $w \in L(M)$?

A. I and IV only B. II and III only C. II, III and IV only D. III and IV only

7. Which one of the following regular expressions represents the language: the set of all binary strings having two consecutive 0s and two consecutive 1s?

A. $(0+1)^*0011(0+1)^*+(0+1)^*1100(0+1)^*$ B. $(0+1)^*(00(0+1)^*11+11(0+1)^*00)(0+1)^*$

C. $(0+1)^*00(0+1)^*+(0+1)^*11(0+1)^*$ D. $00(0+1)^*11+11(0+1)^*00$

8. Which of the following decision problems are undecidable?

I. Given NFAs N_1 and N_2, is $L(N_1) \cap L(N_2) = \Phi$?

II. Given a CFG $G = (N, \Sigma, P, S)$ and a string $x \in \Sigma^*$, does $x \in \Sigma^*, L(G)$?

III. Given CFGs G1 and G2, is $L(G1) = L(G2)$?

IV. Given a TM M, is $L(M) = \Phi$?

A. I and IV only B. II and III only C. III and IV only D. II and IV only

9. The length of the shortest string NOT in the language (over Σ = {a, b}) of the following regular expression is ______________.

a∗b∗(ba)∗a∗

A. 2 B. 3 C. 4 D. 5

10. Let L1 = {w ∈ $\{0,1\}^*$ | w has at least as many occurrences of (110)'s as (011)'s}.

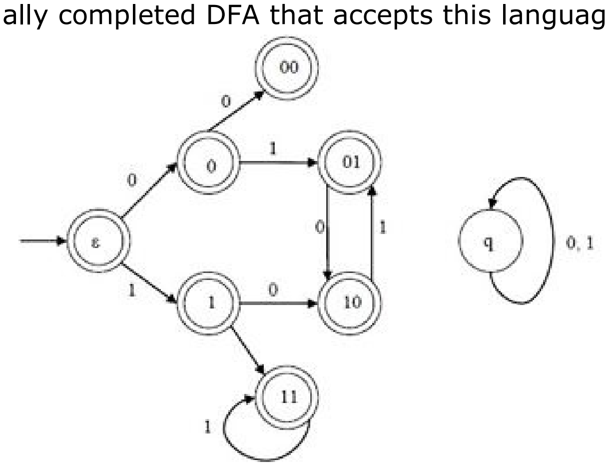
Let L2 = { ∈ $\{0,1\}^*$ | w has at least as many occurrences of (000)'s as (111)'s}.

Which one of the following is TRUE?

A. L_1 is regular but not L_2 B. L_2 is regular but not L_1

C. Both L_1 and L_2 are regular D. Neither L_1 nor L_2 are regular

11. Consider the set of strings on {0,1} in which, *every substring of 3 symbols* has at most two zeros. For example, 001110 and 011001 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially completed DFA that accepts this language is shown below.



The missing arcs in the DFA are

Theory of Computation

A.

	00	01	10	11	q
00	1	0			
01				1	
10	0				
11			0		

B.

	00	01	10	11	q
00		0			1
01		1			
10				0	
11		0			

C.

	00	01	10	11	q
00		1			0
01		1			
10			0		
11		0			

D.

	00	01	10	11	q
00		1			0
01				1	
10	0				
11			0		

12. Given below are two finite slate automata (→indicates the start state and F indicates a final state)

Y:

	a	b
→1	1	2
2 (F)	2	2

Z:

	a	b
→1	2	2
2(F)	1	1

Which of the following represents the product automaton Z x Y?

A.

	a	b
→P	S	R
Q	R	S
R (F)	Q	P
S	Q	P

B.

	a	b
→P	S	Q
Q	R	S
R (F)	Q	P
S	P	Q

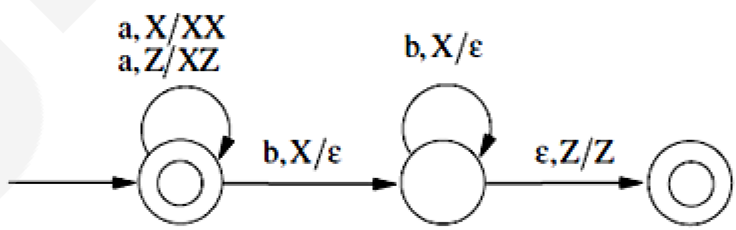
C.

	a	b
→P	Q	S
Q	R	S
R (F)	Q	P
S	Q	P

D.

	a	b
→P	S	Q
Q	S	R
R (F)	Q	P
S	Q	P

13. Consider the transition diagram of a PDA given below with input alphabet $\Sigma = \{a, b\}$ and stack alphabet $\Gamma = \{X, Z\}$. Z is the initial stack symbol. Let L denote the language accepted by the PDA.



Which one of the following is TRUE?

A. $L = \{a^n b^n \mid n \geq 0\}$ and is not accepted by any finite automata

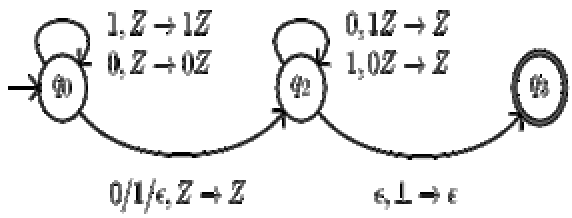
B. $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is not accepted by any deterministic PDA

C. L is not accepted by any Turing machine that halts on every input

D. $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is deterministic context-free

Theory of Computation

14. Consider the NPDA ⟨Q ={q_0, q_1, q_2}, Σ = {0,1}, Γ = {0,1, ⊥}, δ, q_0, ⊥, F = {q_2}⟩, where (as per usual convention) Q is the set of states, Σ is the input alphabet, Γ is the stack alphabet, δ is the state transition function, q_0 is the initial state, ⊥ is the initial stack symbol, and F is the set of accepting states. The state transition is as follows:



Which one of the following sequences must follow the string 101100 so that the overall string is accepted by the automaton?

A. 10110 B. 10010 C. 01010 D. 01001

15. Let L1 be a recursive language. Let L2 and L3 be languages that are recursively enumerable but not recursive. Which of the following statements is not necessarily true?

A. L2 – L1 is recursively enumerable
B. L1 – L3 is recursively enumerable
C. L2 ∩L1 is recursively enumerable
D. L2 ∪L1 is recursively enumerable

16. Consider the following languages.

$L_1 = \{\langle M \rangle \mid M$ takes at least 2016 steps on some input},

$L_2 = \{\langle M \rangle \mid M$ takes at least 2016 steps on all inputs} and

$L_3 = \{\langle M \rangle \mid M$ accepts ε},

where for each Turing machine M, $\langle M \rangle$ denotes a specific encoding of M. Which one of the following is TRUE?

A. L_1 is recursive and L_2, L_3 are not recursive
B. L_2 is recursive and L_1, L_3 are not recursive
C. L_1, L_2 are recursive and L_3 is not recursive
D. L_1, L_2, L_3 are recursive

17. A single tape Turing Machine M has two states q0 and q1, of which q0 is the starting state. The tape alphabet of M is {0, 1, B} and its input alphabet is {0, 1}. The symbol B is the blank symbol used to indicate end of an input string. The transition function of M is described in the following table.

	0	1	B
q0	q1, 1, R	q1, 1, R	Halt
q1	q1, 1, R	q0, 1, L	q0, B, L

The table is interpreted as illustrated below.

The entry (q1, 1, R) in row q0 and column 1 signifies that if M is in state q0 and reads 1 on the current tape square, then it writes 1 on the same tape square, moves its tape head one position to the right and transitions to state q1.

Which of the following statements is true about M?

A. M does not halt on any string in $(0+1)^+$
B. M does not halt on any string in $(00+1)^*$
C. M halts on all strings ending in a 0
D. M halts on all strings ending in a 1

Theory of Computation

18. Consider the following problem X.

Given a Turing machine M over the input alphabet Σ, any state q of M

And a word w ∈Σ*, does the computation of M on w visit the state q?

Which of the following statements about X is correct?

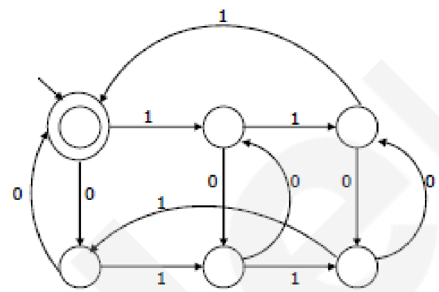
A. X is decidable

B. X is undecidable but partially decidable

C. X is undecidable and not even partially decidable

D. X is not a decision problem

19. The following finite state machine accepts all those binary strings in which the number of 1's and 0's are respectively



A. divisible by 3 and 2

B. odd and even

C. even and odd

D. divisible by 2 and 3

20. Which of the following languages is regular?

A. $\{ww^R \mid w \in \{0,1\}^+\}$

B. $\{ww^R x \mid x, w \in \{0,1\}^+\}$

C. $\{wxw^R \mid x, w \in \{0,1\}^+\}$

D. $\{xww^R \mid x, w \in \{0,1\}^+\}$

ANSWERS

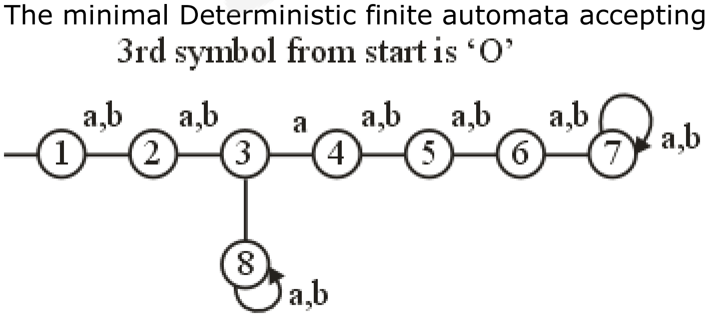
1. Ans. B.
The given Grammar over Σ = {a, b, c} with S as the start symbol is
S → abScT | abcT
T→ bT | b
The minimum length string generated by the grammar is 1:
S→abcT→abcb; hence all variable greater than 1.
Other cases
S → abScT→ ab abScT cT → ab ab abScT cT cT →........→ (ab)n (cT)n.
Here T can generate any number of b's starting with single b.
Hence The language is

$$L = \{(ab)^n cb^{m_1} cb^{m_2} cb^{m_3} cb^{m_4} \ldots\ldots\ldots$$
$$cb^{m_n} \mid m_1, m_2, m_3, m_4 \ldots\ldots\ldots m_n n \geq 1\}$$

2. Ans. A.
Union of context free language is also context free language.
L1 = { a^n b^n c^m| m >= 0 and n >= 0 } and
L2 = { a^m b^nc^n| n >= 0 and m >= 0 }
L3 = L1 ∪ L2 = { $a^nb^nc^m$∪ $a^mb^nc^n$| n >= 0, m >= 0 } is also context free.
L1 says number of a's should be equal to number of b's and L2 says number of b's should be equal to number of c's. Their union says either of two conditions to be true. So it is also context free language.
Intersection of CFG may or may not be CFG.
L3 = L1 ∩ L2 = { $a^nb^nc^n$| n >= 0 } need not be context free

3. Ans. B.
Given L_1 and L_2 are context free languages and R is a regular language.
I. $L_1 \cup L_2$ is context free is CORRECT, context free language are closed under union operation.
II. $\overline{L_1}$ is context free is INCORRECT, context free languages are not closed under complement operation.
III. $L_1 - R$ is Context free is CORRECT.
$L_1 - R = L_1 \cap \overline{R}$, Context free intersection Regular is always Context free.
IV. $L_1 \cap L_2$ is context free is INCORRECT; context free languages are not closed under complement operation.

4. Ans. C.
The Given regular language is L = {w1aw2 | w1,w2 ∈ {a,b}*, |w1|=2,|w2|≥3}
The minimal Deterministic finite automata accepting L is:



ANSWERS

5. Ans. C.
The given table for NFA-ϵ Transition is

δ	ϵ	a	b
$\to q_0$	$\{q_2\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_0\}$	Φ	Φ
q_3	Φ	Φ	$\{q_2\}$

The process is we start with ϵ-closure of q^2 then for each input first take the transition then calculate ϵ-closure

q2 is the start for processing we take ϵ-closure which is {q0, q2} and process "aba"

q2 can reach q0, q1, and q2 after reading "aba".

6. Ans. D.
Is the language represented by regular expression

L(G) is the language generated by context free grammar

L(M) is the language accepted by Turing Machine

I. The problem a given regular expression R and a string w, is is a membership problem. Membership problem is decidable for Finite state machine and regular expression.

II. Given Context free grammar G, is L(G) is φ?, is emptiness problem for context free grammar. Emptiness problem is decidable for CFG by checking usefulness of start symbol.

III. A given context free grammar G, is L(G) is Σ* for some alphabet Σ?, is undecidable problem. We can't check whether L(G) = Σ* or not but rather we can check complement of L(G) is φ. Since context free language are not closed under complement operation L(G) may be language accepted by Turing Machine and we can't check emptiness for Turing machine.

IV. Given a Turing Machine M and a string w, is w Î L(M)?, is a membership problem for TM. Membership problem is not a decidable problem for TM.

7. Ans. B.
A) contains 00 & 11 consecutively which is not the required condition.

C) Doesn't guaranty that both 00 & 11 will be present in the string.

D) Says string should start with 11 & ends with 00 or vice versa.

8. Ans. C.
There is no known algorithm to check whether the language accepted by TM is empty. Similarly there is no algorithm to check whether language CFG's are equivalent.

9. Ans. B.
R = a*b*(ba)*a*

for finding shortest string that is not in language it is better to look strings of length 0, then of length 1 and so on

length0 { ϵ} is in L

length1 {a, b} all belong to L

length2 {aa, ab, ba, bb} all belong to L

length 3 {aaa, aab, aba, abb, baa, bab, bba, bbb} ***bab does not belong to L.***

ANSWERS

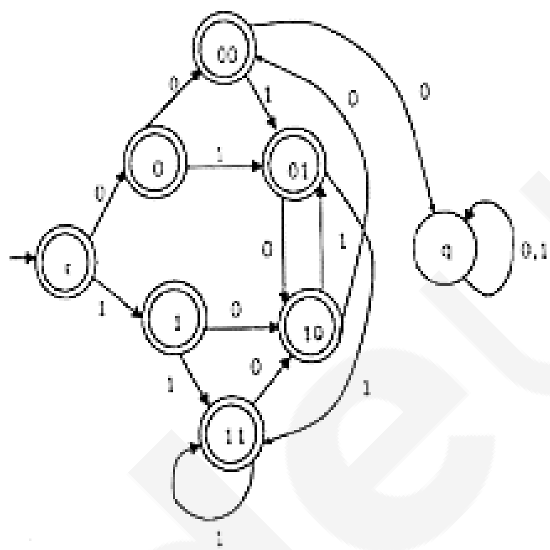
10. Ans. A.

L1 is number of occurrences of 011 are 4 but when we see here 110 is also occurred and the number of occurrence of 110 is 3.

L2 : is follow as 110110110110 in this number of occurrences of 110 is 4 and 011 is 3 which already satisfy the condition

11. Ans. D.

The complete DFA is



12. Ans. A.

Try input 'b' Automation Z goes to a final state Z(F) and so does automation Y δ_0 δ [(1z, 1y), b] = final state.

With output 'b' choices (B), (C) and (D) do not go to a final state.

So the answer is (A)

13. Ans. D.

Given PDA can accept any number of a's by staying at initial state. This PDA can also accept the strings a's followed by b's but equal number of a's and b's by reaching other final state. So $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is deterministic context-free.

14. Ans. B.

In q0 state for '1', a '1' is pushed and for a '0' a '0' is pushed. In q1 state, for a '0' a '1' is popped and for a '1' a '0' is popped. So, the given PDA is accepting all strings of the form x0x'r or x1x'r or xx'r, where x'r is the reverse of the 1's complement of x. The given string 101100 has 6 letters and we are given 5 letter strings. So, x0 is done, with x = 10110. So, x'r = (01001)r = 10010.

Hence option B is correct.

15. Ans. B.

L1 is recursive and L2, L3 are recursively enumerable. So $L1 \cap L3$ and $L2 \cap L3$ can recursively enumerable and also L2 – L1 can recursively enumerable. But we can't say that L1-L3 is also recursively enumerable.

16. Ans. C.

L1 and L2 are recursive. For both of them it is possible to construct halting TM. But L3 is not recursively enumerable hence it is also not recursive. L3 can not accepted by any TM.

ANSWERS

17. Ans. A.

Only Blank symbol, given TM can halt. Remaining all strings, it enters into an infinite loop.

18. Ans. B.

X is undecidable but partially decidable.

We have the TM M. Just make the state q the final state and make all other final states non-final and get a new TM M'. Give input w to M'. If w would have taken M to state q (Yes case of the problem), our new TM M' would accept it. So, the given problem is partially decidable.

If M goes for an infinite loop and never reaches state q (no case for the problem), M' cannot output anything. This problem is the state entry problem, which like word accepting problem and halting problem is undecidable.

19. Ans. A.

This automata accept 10 11 11 0 1

and checking for other input it is clear that it accept number divisible by 3 and 2.

20. Ans. C.

Language L = $\{wxw^R\}x_1 w \in \{0,1\}^+$ is a regular language.

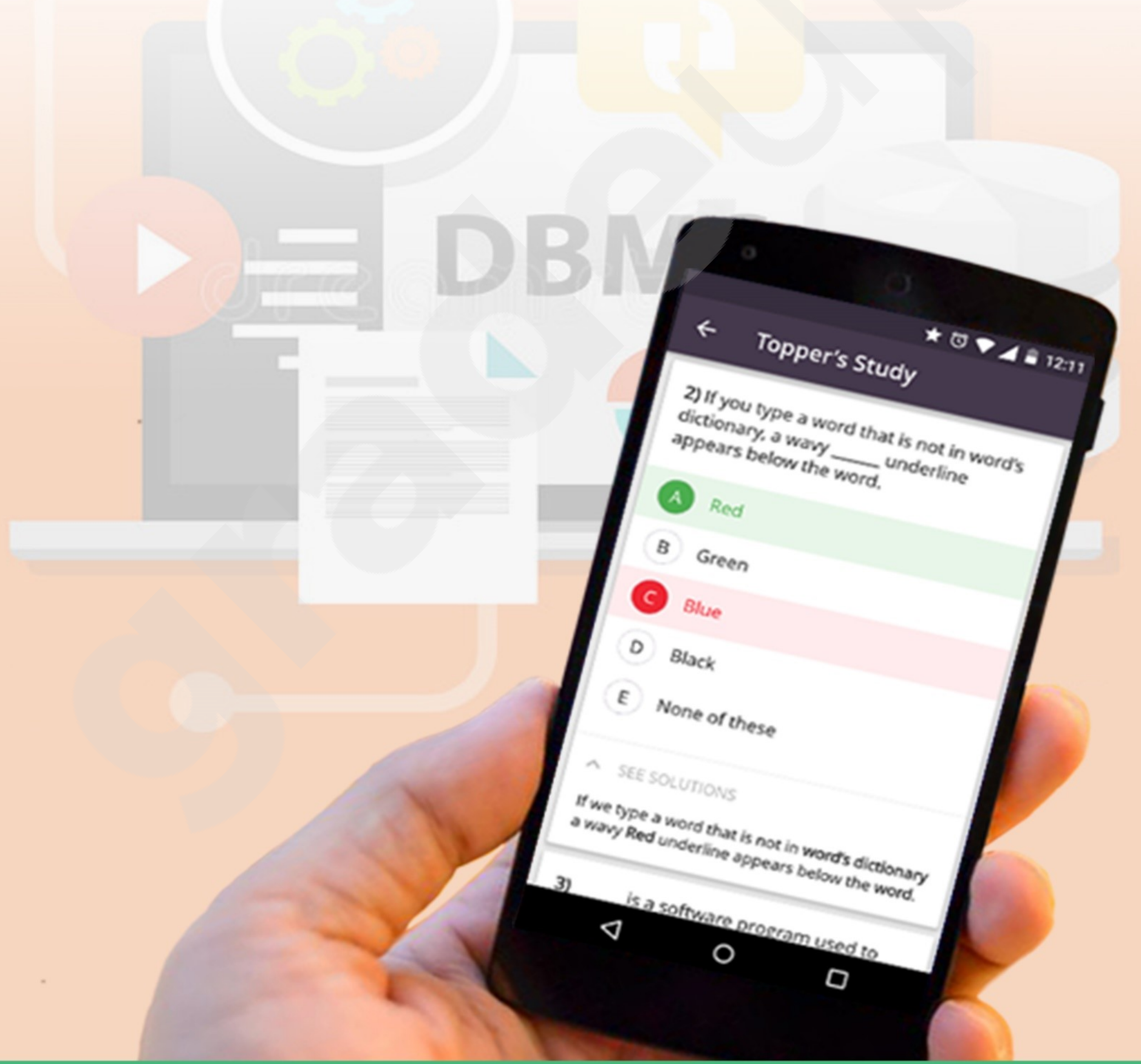
It is a regular language since it is generated by the regular expression

r = 0(0 + 1)* 0 + 1(0 + 1)*1

gradeup

DBMS

Questions & Solutions



DBMS (Databases)

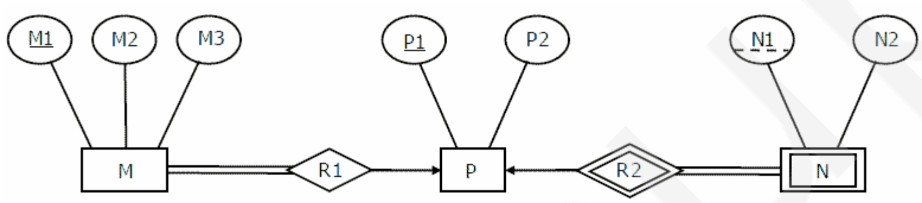
1. Consider an Entity-Relationship (ER) model in which entitysets E_1 and E_2 are connected by an *m* : *n* relationship R_{12}. E_1 and E_3 are connected by a 1 : *n* (1 on the side of E_1 and *n* on the side of E_3) relationship R_{13}.

   E_1 has two single-valued attributes a_{11} and a_{12} of which a_{11} is the key attribute. E_2 has two single- valued attributes a_{21} and a_{22} of which a_{21} is the key attribute. E_3 has two single-valued attributes a_{31} and a_{32} of which a_{31} is the key attribute. The relationships do not have any attributes.

   If a relational model is derived from the above ER model, then the minimum number of relations that would be generated if all the relations are in 3NF is__________.

   A. 2 B. 3 C. 4 D. 5

2. Consider the following ER diagram



   The minimum number of tables needed to represent M, N, P, R1, R2 is

   A. 2 B. 3 C. 4 D. 5

Direction (3-4): Relation *R* has eight attributes *ABCDEFGH*. Fields of *R* contain only atomic values. F={CH → G, A → BC, B → CFH, E → A, F → EG} is a set of functional dependencies (FDs) so that F^+ is exactly the set of FDs that hold for R.

3. How many candidate keys does the relation R have?

   A. 3 B. 4 C. 5 D. 6

4. The relation R is

   A. in 1NF, but not in 2NF B. in 2NF, but not in 3NF

   C. in 3NF, but not in BCNF D. in BCNF

5. A database of research articles in a journal uses the following schema. (VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, YEAR, PRICE)

   The primary key is (VOLUME, NUMBER, STARTPAGE, ENDPAGE) and the following functional dependencies exist in the schema.

   (VOLUME, NUMBER, STARTPAGE, ENDPAGE) → TITLE

   (VOLUME, NUMBER → YEAR

   (VOLUME, NUMBER, STARTPAGE, ENDPAGE) → PRICE

   The database is redesigned to use the following schemas.

   (VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, PRICE) (VOLUME, NUMBER, YEAR)

   Which is the weakest normal form that the new database satisfies, but the old one does not?

   A. 1NF B. 2NF C. 3NF D. BCNF

6. R(A,B,C,D) is a relation. Which of the following does not have a lossless join, dependency preserving BCNF decomposition?

   A. A →B, B →CD B. A →B, B →C, C →D

   C. AB →C, C →AD D. A →BCD

DBMS (Databases)

7. Which of the following tuple relational calculus expression(s) is/are equivalent to $\forall t \in r(P(t))$?

I. $\neg\exists t \in r(P(t))$

II. $\exists t \notin r(P(t))$

III. $\neg\exists t \in r(\neg P(t))$

IV. $\exists t \notin r(\neg P(t))$

A. I only B. II only C. III only D. III and IV only

8. Consider the following schedule for transactions T1, T2 and T3:

T1	T2	T3
Read(X)		
	Read(Y)	
		Read(Y)
	Write(Y)	
Write(X)		
		Write(X)
	Read(X)	
	Write(X)	

Which one of the schedules below is the correct serialization of the above?

A. T1 → T3 → T2
B. T2 → T1 → T3
C. T2 → T3 → T1
D. T3 → T1 → T2

9. Consider the following schedules involving two transactions. Which one of the following statements is **TRUE**?

$S_1 : r_1(X); r_1(Y); r_2(X); r_2(Y); w_2(Y); w_1(X)$

$S_2 : r_1(X); r_2(X); r_2(Y); w_2(Y); r_1(Y); w_1(X)$

A. Both S_1 and S_2 are conflict serializable.
B. S_1 is conflict serializable and S_2 is not conflict serializable.
C. S_1 is not conflict serializable and S_2 is conflict serializable.
D. Both S_1 and S_2 are not conflict serializable.

10. Consider the transactions T1, T2, and T3 and the schedules S1 and S2 given below:

T1: r1(X); r1(Z); w1(X); w1(Z)
T2: r2(Y); r2(Z); w2(Z)
T3: r3(Y); r3(X); w3(Y)
S1:r1(X);r3(Y);r3(X);r2(Y);r2(Z);w3(Y);w2(Z);r1(Z);w1(X);w1(Z)
S2:r1(X);r3(Y);r2(Y);r3(X);r1(Z);r2(Z);w3(Y);w1(X);w2(Z);w1(Z)

Which one of the following statements about the schedules is TRUE?

A. Only S1 is conflict-serializable.
B. Only S2 is conflict-serializable.
C. Both S1 and S2 are conflict-serializable.
D. Neither S1 nor S2 is conflict-serializable.

11. Let R(A,B,C,D,E,F,G) be a relational schema in which the following functional dependencies are known to hold:

AB→CD, DE→F, C→E, F→C and B→G.

Then the relation schema R is in

A. In BCNF
B. In 3NF, but not in BCNF
C. In 2NF, but not in 3NF
D. Not in 2NF

DBMS (Databases)

12. A B^+ - tree index is to be built on the Name attribute of the relation STUDENT. Assume that all student names are of length 8 bytes, disk blocks are of size 512bytes, and index pointers are of size 4 bytes. Given this scenario, what would be the best choice of the degree (i.e. the number of pointers per node) of the B^+ -tree?

A. 16 B. 42 C. 43 D. 44

13. Here, Record Size= 32 Bytes
Key Size= 6 Bytes .
Consider a file of 16384 records. Each record is 32 bytes long and its key field is of size 6 bytes. The file is ordered on a non-key field, and the file organization is unspanned. The file is stored in a file system with block size 1024 bytes, and the size of a block pointer is 10 bytes. If the secondary index is built on the key field of the file, and a multi-level index scheme is used to store the secondary index, the number of first-level and second-level blocks in the multi-level index are respectively

A. 8 and 0 B. 128 and 6 C. 256 and 4 D. 512 and 5

14. Consider the following relation
Cinema (*theater*, *address*, *capacity*)
Which of the following options will be needed at the end of the SQL query SELECT P1.*address* FROM Cinema P1
such that it always finds the addresses of the alters with maximum capacity?
A. WHEREP1.*capacity*>=All (select P2.*capacity* from Cinema P2)
B. WHEREP1.*capacity*>=Any (select P2.*capacity* from Cinema P2)
C. WHEREP1.*capacity*> All (select max (P2.*capacity*) from Cinema P2)
D. WHEREP1.*capacity*> Any (select max (P2.*capacity*) from Cinema P2)

Direction (15-16): Consider the following relations A, B and C:

A

Id	Name	Age
12	Arun	60
15	Shreya	24
99	Rohit	11

B

Id	Name	Age
15	Shreya	24
25	Hari	40
98	Rohit	20
99	Rohit	11

C

Id	Phone	Area
10	2200	02
99	2100	01

15. How many tuples does the result of the following relational algebra expression contain? Assume that the schema of A∪B is the same as that of A.
$(A \cup B) \bowtie_{A.Id > 40 \vee C.Id < 15} C$

A. 7 B. 4 C. 5 D. 9

16. Consider the table **employee**(<u>empId</u>, name, department, salary) and the two queries Q_1,Q_2 below. Assuming that department 5 has more than one employee, and we want to find the employees who get higher salary than anyone in thedepartment 5, which one of the statements is **TRUE** for any arbitrary employee table?

DBMS (Databases)

Q_1: Select e.empId
From employee e
Where not exists
(Select * From employee s where s.department = "5" and s.salary>=e.salary)
Q_2: Select e.empId
From employee e
Where e.salary>Any
(Select distinct salary From employee s Where s.department = "5")

A. Q_1 is the correct query
B. Q_2 is the correct query
C. Both Q_1 and Q_2 produce the same answer.
D. Neither Q_1 nor Q_2 is the correct query

17. Consider the following database table named water schemes:

Water schemes		
Scheme no	District name	Capacity
1	Ajmeer	20
1	Bikaner	10
2	Bikaner	10
3	Bikaner	20
1	Churu	10
2	Churu	20
1	Dungargarh	10

The number of tuples returned by the following SQL query is ______.
with total(name, capacity) as
select district_name, sum(capacity)
from water_schemes
group by district_name
with total_avg (capacity) as select avg(capacity)
from total
select name
from total, total_avg
where total.capacity ≥ total_avg.capacity

A. 1
B. 2
C. 3
D. 4

18. Let R and S be relational schemes such that R = {a, b, c} and S = {c}. Now consider the following queries on the
Database:

I. $\pi_{R-S}(r) - \pi_{R-S}(\pi_{R-S}(r) \times s - \pi_{R-S,S}(r))$

II. $\{t \mid t \in \pi_{R-S}(r) \wedge \forall u \in s(\exists v \in r(u = v[s] \wedge t = v[R-S]))\}$

III. $\{t \mid t \in \pi_{R-S}(r) \wedge \forall v \in r(\exists u \in s(u = v[s] \wedge t = v[R-S]))\}$

IV. Select *R.a, R.b*
from *R, S*
where *R.c* = *S .c*

Which of the above queries are equivalent?

A. I and II
B. I and III
C. II and IV
D. III and IV

DBMS (Databases)

19. Let R and S be two relations with the following schema

R(P, Q, R1, R2, R3)

S (P, Q, S1, S2)

where {P, Q} is the key for both schemas. Which of the following queries are equivalent?

I. $\pi_P(R \bowtie S)$

II. $\pi_P(R) \bowtie \pi_P(S)$

III. $\pi_P(\pi_{P,Q}(R) \cap \pi_{P,Q}(S))$

IV. $\pi_P(\pi_{P,Q}(R) - (\pi_{P,Q}(R) - \pi_{P,Q}(S)))$

A. Only I and II

B. Only I and III

C. Only I, II and III

D. Only I, III and IV

20. Consider the following relational schema.

Students(rollno: integer, sname: string)

Courses(courseno: integer, cname: string)

Registration(rollno: integer, courseno: integer, percent: real)

Which of the following queries are equivalent to this query in English?

“Find the distinct names of all students who score more than 90% in the course numbered 107”

(I) SELECT DISTINCT S.sname

FROM Students as S, Registration as R

WHERE R.rollno=S.rollno AND R.courseno=107 AND R.percent >90

(II) Î $_{sname}(\sigma_{courseno= 107 \wedge percent>90}$ (Registration ⋈ Students))

(III) {T | ∃S∈Students, ∃R∈Registration (S.rollno = R.rollno ∧ R.courseno=107 ∧R.percent>90 ∧T.sname=S.sname)}

(IV) {<S_N> | ∃S_R∃R_P (<S_R, S_N>∈Students ∧ <S_R, 107, R_P>∈Registration ∧ R_P>90)}

A. I, II, III and IV

B. I, II and III only

C. I, II and IV only

D. II, III and IV only

ANSWERS

1. Ans. C.

Entity E1

a1 a12

----

a11 is key

Entity E2

a21 a22

----

a22 is key

Entity E3

a31 a32

----

a31 is key

R12 is m:n Relationship between E1 and E2

R12

a11 a22

(a11, a22) is key.

R13 is 1:n Relationship between E1 and E3

R13

a11 a31

.

.

a31 is the key

We need minimum no. of tables.

Can we remove any of the above tables without loosing information and keeping the relations in 3NF?

We can combine R13 and E2 into one.

a11 a31 a22

(a11, a31, a22) is key.

The relation is still in 3NF as for every functional dependency X -> A, one of the following holds

1) X is a super key or

2) A-X is prime attribute

ANSWERS

2. Ans. B.

Three tables will be required to the entities M, P and weak entity set N.

Table N, attributes N_1, N_2

N, R2 can be merged into single table because N is a weak entity set.

Also, (M, R1) can be merged into single table due to total participation of M and relationship being M:1.

So total no of tables needed= 3

3. Ans. B.

There are four keys: (1) AD, ED, BD, RD we can find key by find closure

so

{AD}* = {A, B, C, D, E, F, G, H}

{ED}* = {A, B, C, D, E, F, G, H}

{BD}* = {A, B, C, D, E, F, G, H}

{FD}* = {A, B, C, D, E, F, G, H}

4. Ans. A.

Relation is in INF because it is atomic

Relation is not in 2NF because prime attributes of

Relation is: {A, B, D, E, F}

$A \to BC \Rightarrow$ written as (a) $A \to B$

wrong (b) $A \to C$

So (b) $A \to C$

proper subset of candidate key → Non prime attribute. Violates definition of 2NF, that every non prime attribute should device from candidate key or not drive from prime attribute.

(c) Wrong answer → because relation is not in 2NF so it is wrong option.

(d) Wrong answer → because relation is not is 2NF. So not in 3NF & BCNF.

5. Ans. B.

candidate key is (volume, number, start page, end page)

(Volume number) → year is a partial dependency. So original table is in 1NF but not in 2NF

6. Ans. C.

first, we know what is Lossless -join decomposition and Dependency preserving.

- **Lossless-Join Decomposition:**
- Decomposition of R into R1 and R2 is a lossless-join decomposition if at least one of the following functional dependencies are in F+ (Closure of functional dependencies)
- R1 ∩ R2 → R1
- OR
- R1 ∩ R2 → R2
- **dependency preserving :**
- Decomposition of R into R1 and R2 is a dependency preserving decomposition if closure of functional dependencies after decomposition is same as closure of of FDs before decomposition.
- A simple way is to just check whether we can derive all the original FDs from the FDs present after decomposition.

Now take each.

A) A->B, B->CD

R1(AB) and R2(BCD)

B is the key of second and hence decomposition is lossless.

ANSWERS

B) A->B, B->C, C->D

R1(AB) , R2(BC), R3(CD)

B is the key of second and C is the key of third, hence lossless.

C) AB->C, C->AD

R1(ABC), R2(CD)

C is key of second, but C->A violates BCNF condition in ABC as C is not a key. We cannot decompose ABC further as AB->C dependency would be lost.

D) A ->BCD

Already in BCNF.

Therefore, Option C AB->C, C->AD is the answer.

7. Ans. C.

$\forall t \in r(P(t))$ is equivalent to the relational calculus expression $\exists t \in r(-P(t))$.

The given statement means for all tuples from r, P is true. Ill means there does not exist a tuple in r where P is not true. Both are equivalent.

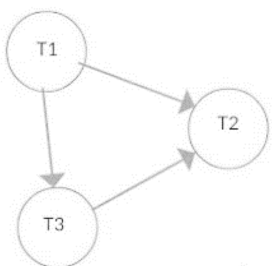
IV is not correct as it as saying that there exist a tuple, not in r for which P is not true, which is not what the given expression means.

8. Ans. A.

Answer = **option A**

create precedence graph and apply Topological sort on it to obtain

T1→T3→T2



9. Ans. C.

S_1 :

$r_1(Y)$, $w_2(Y)$ are conflict, hence $T_1 < T_2$.

$r_2(X)$, $w_1(x)$ are conflict, hence $T_2 < T_1$.

There is no serial schedule which satisfies both the above rules.

Hence S is not conflict serializable.

S_2 :

$r_2(X)$, $w_1(x)$ are conflict, hence $T_2 < T_1$.

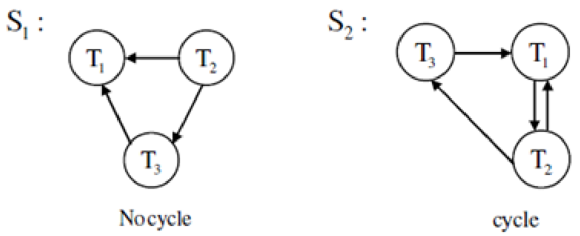
w_2 (Y), r_1 (Y) are conflict, hence $T_2 < T_1$

There is a series schedule that satisfies $T_2 < T_1$ hence S is conflict serializable.

10. Ans. A.

For conflict serializability of a schedule(which gives same effect as a serial schedule) we should check for conflict operations, which are Read-Write, Write-Read and Write-Write between each pair of transactions, and based on those conflicts we make a precedence graph, if the graph contains a cycle, it's not a conflict serializable schedule. Precedence graph for S1 and S2 are as follows:

ANSWERS



From these graphs, it is clear that S1 is conflict-serializable.

11. Ans. D.
AB can determine any attribute thus AB will be candidate key, but there is a partial dependency, as G only depends on B. Thus the relation is not in 2nd Normal form because to be in 2nd Normal form the relation must be in 1st NF and all non-key attributes must be fully functionally dependent on the primary key.

12. Ans. C.
Size of 1 record = 8 + 4 = 12
Let the order be N.
No. of index values per block = N - 1
(N - 1) 12 + 4 = 512
12N - 12 + 4 = 512
16N = 1009
N = 43.3333

13. Ans. C.
When secondary index is built on the key field of the file, and a multilevel scheme is used to store the secondary index, then the no. of first level and second level index are 256 and 4.
Here, Record Size= 32 Bytes
Key Size= 6 Bytes
Block pointer Size= 10 bytes
Block Size= 1024 B
Block Factor for index file= 64
Block factor for DB= 32
then no of block at first level in secondary index= 16384/64= 256
No of block at second level in seconary index= 256/64= 4

14. Ans. A.
Option A is correct. When the ALL condition is followed by a list, the optimizer expands the initial condition to all elements of the list and strings them together with AND operators. When the **ANY** condition is followed by a list, the optimizer expands the initial condition to all elements of the list and strings them together with OR operators.
Logical Operators

Operator	Description	Example
&&	Called Logical AND operator. If both the operands are non-zero, then the condition becomes true.	(A && B) is false.
||	Called Logical OR Operator. If any of the two operands is non-zero, then the condition becomes true.	(A || B) is true.

ANSWERS

15. Ans. A.

The final table is

AUB.Id	Name	Age	C.Id	Phone	Area
12	Arun	60	10	2200	02
15	Shreya	24	10	2200	02
25	Hari	40	10	2200	02
98	Rohit	20	10	2200	02
98	Rohit	20	99	2100	01
99	Rohit	11	10	2200	02
99	Rohit	11	99	2100	01

16. Ans. B.

Since Q_1 consists "not exists", it produces undesired results in certain conditions. Ex: If department = 4 and s.salary>=e.salary, or department = 4 and s.salary<= e.salary, condition is always failed hence inner query produces non empty set.

This makes "not selected for output. In Q_2, inner query produces salaries for department 5, > Any operator perfectly produces the desired results.

17. Ans. B.

With clause is used to used the result of a query as some table at a particuar instance.

First with clause will produce the sum of total capacity on the basis of districts.

Second With clause will produce the avg of all the capacity

Actula Query will output those district where total capacity is greater than average capacity.

Two names Bikaner & churu will be selected.

18. Ans. A.

Query ! and 2 are equivalent since query 2 is the tuple alzebra for division operator and Query 1 is the realtional alzebra for division operator. whereas, Query 3 calculates the natural join of R & S.

19. Ans. D.

In I value P_s from natural join of R and S are selected.

In III, all Ps from intersection of (P, Q) pairs present in R and S.

IV is also equivalent to III because (R – (R – S)) = R ∩ S.

II is not equivalent as it may also include Ps where Qs are not same in R and S

20. Ans. A.

Find the distinct names of all students who score more than 90% in the course no. 107.

1. SQL query

Condition would give all S. name having score > 90 and attending course no. 107 and DISTINCT sname will give distinct student names.

TRUE

2. Relational Algebra

Î sname gives projection of all students meeting the condition and 'п' gives DISTINCT value.

TRUE

3. Tuple Calculus

Gives DISTINCT student name having score > 90 and course no. is 107. TRUE

4. Domain Calculus

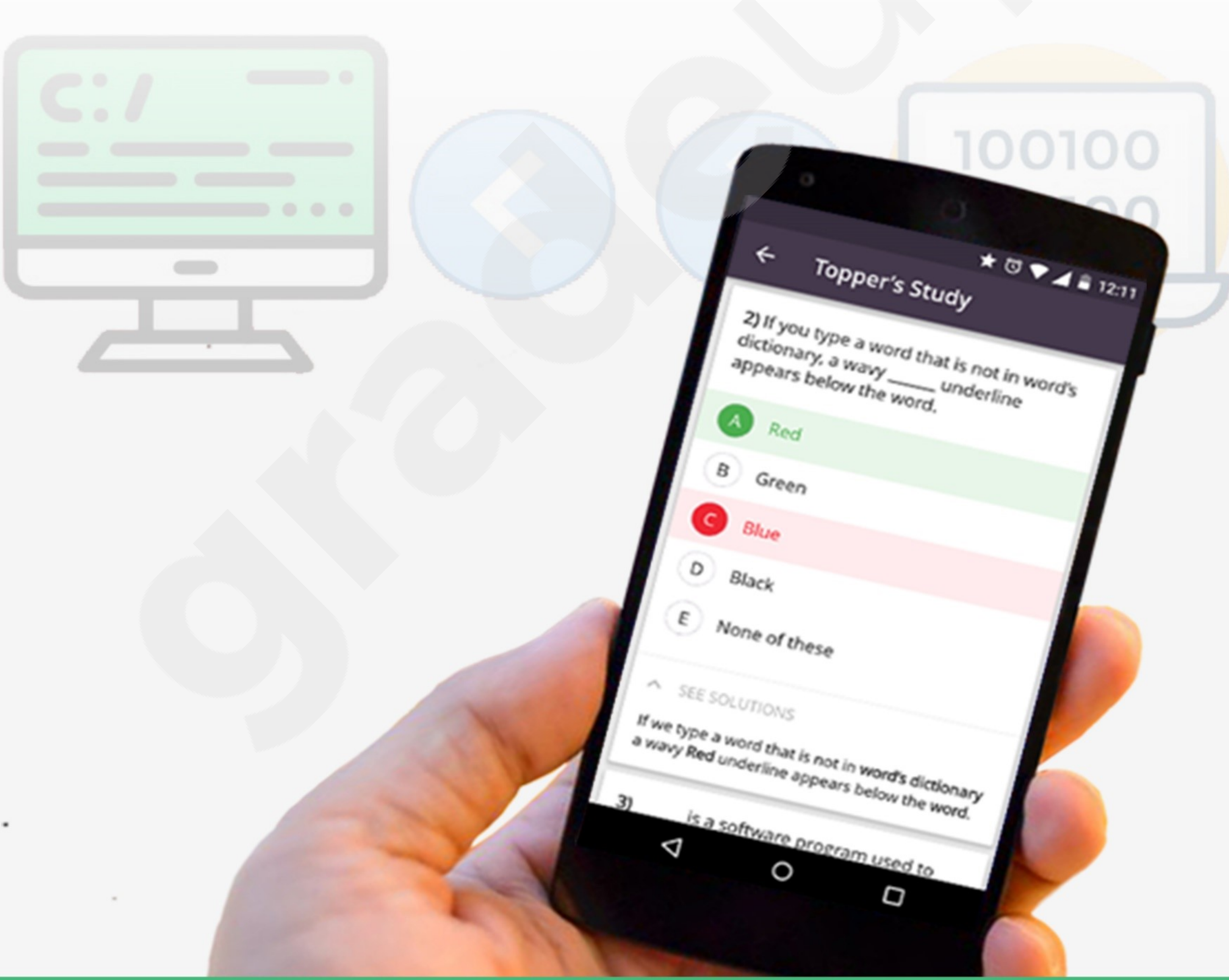
Domain calculus is equivalent to relational algebra and provide distinct value for the query.

TRUE

gradeup

Compiler Design

Questions & Solutions



Compiler Design

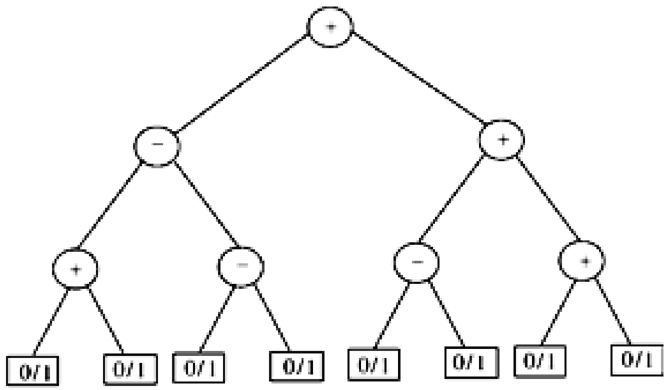
1. A student wrote two context-free grammars G1 and G2 for generating a single C-like array declaration. The dimension of the array is at least one. For example,
int a[10][3];
The grammars use D as the start symbol, and use six terminal symbols int ; id [ ] num.
Grammar G1
$D \to int\ L;$
$L \to id\ [E$
$E \to num]$
$E \to num]\ [E$
Grammar G2
$D \to int\ L;$
$L \to id\ E$
$E \to E[num]$
$E \to [num]$
Which of the grammars correctly generate the declaration mentioned above?
A. Both G1 and G2
B. Only G1
C. Only G2
D. Neither G1 nor G2

2. Consider the following code segment.
$x = u - t;$
$y = x * v;$
$x = y + w;$
$y = t - z;$
$y = x * y;$
The minimum number of total variables required to convert the above code segment to static single assignment form is __________.
A. 10
B. 6
C. 7
D. 9

3. Consider the intermediate code given below.
(1) i = 1
(2) j = 1
(3) t1 = 5 ∗ i
(4) t2 = t1 + j
(5) t3 = 4 ∗ t2
(6) t4 = t3
(7) a[t4] = -1
(8) j = j + 1
(9) if j<=5 goto (3)
(10) i=i+1
(11) if i<5 goto (2)
The number of nodes and edges in the control-flow-graph constructed for the above code, respectively, are
A. 4 and 5
B. 6 and 7
C. 5 and 5
D. 7 and 8

4. Consider the expression tree shown. Each leaf represents a numerical value, which can either be 0 or 1. Over all possible choices of the values at the leaves, the maximum possible value of the expression represented by the tree is ______.

Compiler Design



A. 5 B. 6 C. 7 D. 8

5. Among simple LR (SLR), canonical LR, and look-ahead LR (LALR), which of the following pairs identify the method that is very easy to implement and the method that is the most powerful, in that order?
A. SLR, LALR
B. Canonical LR, LALR
C. SLR, canonical LR
D. LALR, canonical LR

6. Which one of the following is TRUE at any valid state in shift-reduce parsing?
A. Viable prefixes appear only at the bottom of the stack and not inside
B. Viable prefixes appear only at the top of the stack and not inside
C. The stack contains only a set of viable prefixes
D. The stack never contains viable prefixes

7. Consider the augmented grammar G shown below:

G: $S' \rightarrow S$
$S \rightarrow abS \mid ScS \mid d \mid e$

Find the total number of inadequate (conflict) states in the state diagram using LR(1) parser for the grammar G.
A. 1 B. 2 C. 3 D. 4

8. Consider line number 3 of the following C- program.
```
int main ( ) { /* Line 1 */
int I, N; /* Line 2 */
for (I = 0, I < N, I++); /* Line 3 */
}
```
Identify the compiler's response about line 3 while creating the object-module
A. No compilation error
B. Only a lexical error
C. Only syntactic errors
D. Both lexical and syntactic errors

9. Consider the following grammar *G*

$S \rightarrow F \mid H$
$F \rightarrow p \mid c$
$H \rightarrow d \mid c$

where S,F, and Hare non-terminal symbols , p, d, and care terminal symbols. Which of the following statement (s) is / are correct?
S1. LL(1) can parse al strings that are generated using grammar G
S2. LR(1) can parse all strings that are generated using grammar G
A. Only S1
B. Only S2
C. Both S1 and S2
D. Neither S1 nor S2

Compiler Design

10. What is the maximum number of reduce moves that can be taken by a bottom-up parser for a grammar with no epsilon-and unit-production (i.e., of type A -> ∊ and A -> a) to parse a string with n tokens?
A. n/2   B. n – 1   C. 2n – 1   D. 2^n

11. Consider the following two sets of LR(1) items of an LR(1) grammar.
Set-1:
X → c.X, c/d
X → .cX, c/d
X → .d, c/d
Set-2:
X → c.X, $
X →.cX, $
X →.d, $
Which of the following statements related to merging of the two sets in the corresponding LALR parser is/are FALSE?
(1). Cannot be merged since look heads are different.
(2). Can be merged but will result in S-R conflict.
(3). Can be merged but will result in R-R conflict.
(4). Cannot be merged since goto on c will lead to two different sets.
A. 1 only   B. 2 only   C. 1 and 4 only   D. 1, 2, 3 and 4

12. An LALR (1) parser for a grammar G can have shift-reduce (S-R) conflicts if and only if
A. The SLR(1) parser for G has S-R conflicts
B. The LR(1) parser for G has S-R conflicts
C. The LR(0) parser for G has S-R conflicts
D. The LALR(1) parser for G has reduce-reduce conflicts

13. Consider the following Syntax Directed Translation Scheme (SDTS), with non-terminals {S, A} and terminals {a, b}.
S → aA {print 1}
S → a {print 2}
A → Sb {print 3}
Using the above SDTS, the output printed by a bottom-up parser, for the input aab is:
A. 1 3 2   B. 2 2 3   C. 2 3 1   D. syntax error

14. The attributes of three arithmetic operators in some programming language are given below.

Operator	Precedence	Associativity	Arity
+	High	Left	Binary
–	Medium	Right	Binary
*	Low	Left	Binary

The value of the expression $2-5+1-7*3$ in this language is ___________.
A. 3   B. 6   C. 9   D. 12

15. Consider the following translation scheme.
S →ER
R →*E {print ('*');}R|ε
E →F + E {print ('+');}|F
F →(S)|id {print (id.value);}
Here id is a token that represents an integer and id.value represents the corresponding integer value. For an input '2 * 3 +4', this translation scheme prints
A. 2 * 3 +4   B. 2 * +3 4   C. 2 3 * 4 +   D. 2 3 4+*

Compiler Design

16. Consider the following intermediate program in three address code

$p = a - b$

$q = q * c$

$p = u * v$

$q = p + q$

Which one of the following corresponds to a static single assignment form of the above code?

A. $p_1 = a - b$, $q_1 = p_1 * c$, $p_1 = u * v$, $q_1 = p_1 + q_1$

B. $p_3 = a - b$, $q_4 = p_3 * c$, $p_4 = u * v$, $q_5 = p_4 + q_4$

C. $p_1 = a - b$, $q_1 = p_2 * c$, $p_3 = u * v$, $q_2 = p_4 + q_3$

D. $p_1 = a - b$, $q_1 = p_2 * c$, $p_2 = u * v$, $q_2 = p + q$

17. Consider the following grammar.

$P \rightarrow xQRS$

$Q \rightarrow yz | z$

$R \rightarrow w | \epsilon$

$S \rightarrow y$

What is FOLLOW (Q)?

A. {R}　B. {w}　C. {w, y}　D. {w, $}

18. Match the following according to input (from the left column) to the complier phase (in the right column) that processes it.

Column-1		Column-2	
P.	Syntax tree	i.	Code generator
Q.	Character stream	ii.	Syntax analyzer
R.	Intermediate representation	iii.	Semantic analyzer
S.	Token stream	iv	Lexical analyzer

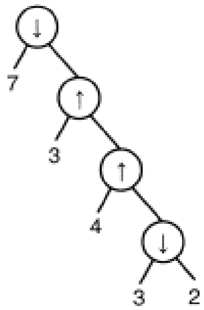
A. P-(ii), Q-(iii), R-(iv), S-(i)

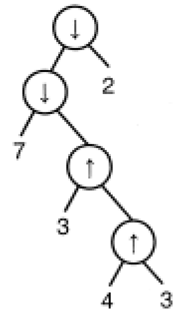
B. P-(ii), Q-(i), R-(iii), S-(iv)

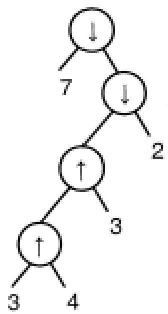
C. P-(iii), Q-(iv), R-(i), S-(ii)

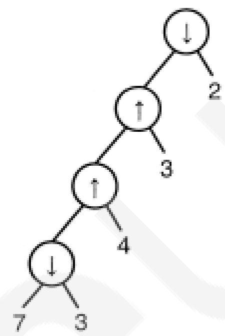
D. P-(i), Q-(iv), R-(ii), S-(iii)

19. Consider two binary operators '↑' and '↓' with the precedence of operator ↓ being lower than that of the operator ↑. Operator ↑ is right associative while operator ↓ is left associative. Which one of the following represents the parse tree for expression (7 ↓ 3 ↑ 4 ↑ 3 ↓ 2)?

A. 

B. 

C. 

D. 

20. For the grammar below, a partial LL(1) parsing table is also presented along with the grammar. Entries that need to be filled are indicated as E1, E2, and E3. ϵ is the empty string, \$ indicates end of input, and, | separates alternate right-hand sides of production.

$S \rightarrow a\,A\,b\,B \mid b\,A\,a\,B \mid \varepsilon$
$A \rightarrow S$
$B \rightarrow S$

	a	b	\$
S	**E1**	**E2**	$S \rightarrow \varepsilon$
A	$A \rightarrow S$	$A \rightarrow S$	error
B	$B \rightarrow S$	$B \rightarrow S$	**E3**

A. FIRST(A) = {a, b, ε} = FIRST(B)
FOLLOW(A) = {a, b}
FOLLOW(B) = {a, b, \$}

B. FIRST(A) = {a, b, \$}
FIRST(B) = {a, b, ε}
FOLLOW(A) = {a, b}
FOLLOW(B) = {\$}

C. FIRST(A) = {a, b, ε} = FIRST(B)
FOLLOW(A) = {a, b}
FOLLOW(B) = ∅

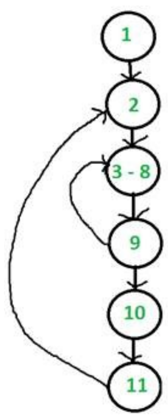
D. FIRST(A) = {a, b} = FIRST(B)
FOLLOW(A) = {a, b}
FOLLOW(B) = {a, b}

ANSWERS

1. Ans. A.
Both G1 & G2 generates the string: int a[10] [3];
You may use LMD, RMD or parse tree to derive the string.

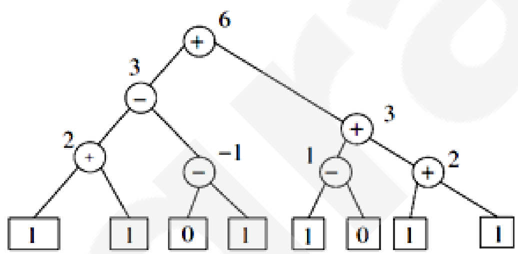
2. Ans. A.
x1 = u - t;
y1 = x1 * v;
x2 = y1 + w;
y2 = t - z;
y3 = x * y2;
So, we require total 10 variables in SSA form of the given code.

3. Ans. B.



The above is the control flow graph.

4. Ans. B.



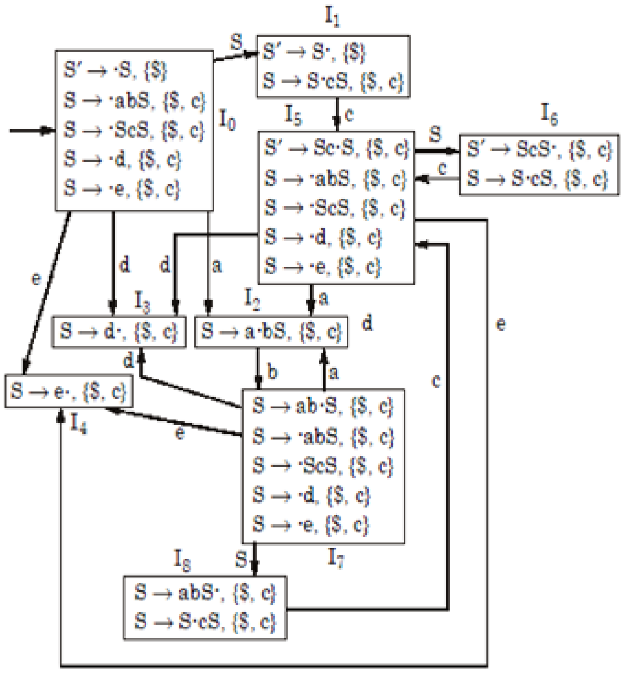
So as per the above tree where leaves have been given the values, the maximum possible value of the expression represented by the tree is 6.

5. Ans. C.
SLR parser is a type of LR parser with small parse tables and a relatively simple parser generator algorithm. Canonical LR parser or LR (1) parser is an LR (k) parser for k=1, i.e. with a single look ahead terminal. It can handle all deterministic context-free languages. LALR parser or Look-Ahead LR parser is a simplified version of a canonical LR parser.

6. Ans. C.
The prefixes of right sentential forms that can appear on the stack of a shift-reduce parser are called viable prefixes. By definition, a viable prefix is a prefix of a right sentential form that does not continue past the right end of the rightmost handle of that sentential form.

ANSWERS

7. Ans. B.



In above LALR (1), I_6 and I_8 contains SR conflicts.
∴ 2 states are conflicts states.

8. Ans. C.
for syntax mismatch in the given program. for has three statements separated by semicolon. It produces syntax error.

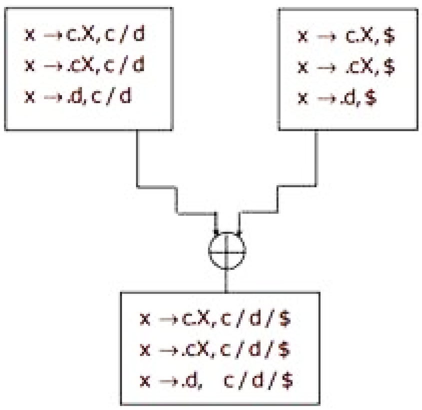
9. Ans. D.
The given grammar is ambiguous as there are two possible leftmost derivations for string "c".
First Leftmost Derivation
S → F
F → c
Second Leftmost Derivation
S → H
H → c
An Ambiguous grammar can neither be LL (1) nor LR (1) , so the Result is Neither S1 nor S2

10. Ans. B.
Given in the question, a grammar with no epsilon- and unit-production (i.e., of type A -> ϵ and A -> a).
Suppose the string is abcd. (n = 4)
We can write the grammar which accepts this string as follows:
S->aB
B->bC
C->cd
The Right Most Derivation for the above is:
S -> aB (Reduction 3)
-> abC (Reduction 2)
-> abcd (Reduction 1)
We can see here that no production is for unit or epsilon. Hence 3 reductions here.We can get less number of reductions with some other grammar which also does’t produce unit or epsilon productions,
S->abA
A-> cd

ANSWERS

The Right Most Derivation for the above as:
S -> abA (Reduction 2)
-> abcd (Reduction 1)
Hence 2 reductions.
But we are interested in knowing the maximum number of reductions which comes from the 1st grammar. Hence total 3 reductions as maximum, which is (n – 1) as n = 4 here.
Thus, Option B

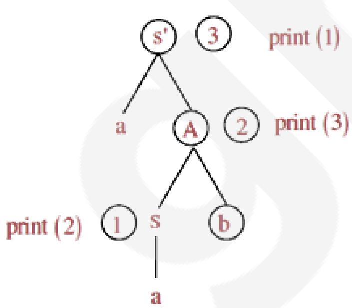
11. Ans. D.



(1). Merging of two states depends on core part (production rule with dot operator), not on lookaheads.
(2). The two states are not containing Reduce item, so after merging, the merged state cannot contain any S-R conflict.
(3). As there is no Reduce item in any of the state, so can't have R-R conflict.
(4). Merging of stats does not depend on further goto on any terminal.
So all statements are false.

12. Ans. B.
An LA LR(1) passer for a grammar G can have shift-reduce (SR) conflicts if and only if the LR(1) passer for G has S-R conflicts.

13. Ans. C.



14. Ans. C.
2-5+1-7*3
2-(5+1) -7*3
2-6-7*3
2-(6-7)*3
2-(-1)*3
2+1*3
3*3
9

ANSWERS

15. Ans. D.
As per the translation scheme given by
E → F + E {print ('+');}|F, the translation scheme prints of input '2 * 3 +4', is
2 3 4+*.
Hence option (D) is correct.

16. Ans. B.
According to Static Single Assignment
- A variable cannot be used more than once in the LHS
- A variable should be initialized atmost once.

Now looking at the given options
1. a – code violates condition 1 as p1 is initialized again in this statement: p1 = u * v
2. c- code is not valid as $q_1 = p_2 * c$, $q_2 = p_4 + q_3$ – In these statements p_2, p_4, q_3 are not initialized anywhere
3. d- code is invalid as $q_2 = p + q$ is incorrect without moving it to register

Therefore, **option B** is only correct option.

17. Ans. C.
FOLLOW(Q) is FIRST(R) hence
FIRST (R)={w,ε}
We add 'w' in FOLLOW(Q) and for ϵ we calculate FIRST(S)
FIRST(S) ={y}
FOLLOW(Q) is {w,y}

18. Ans. C.
Lexical Analysis phase processes character stream and generates tokens, e.g. identifier or keywords.
Tokens are processed by Syntax analysis analyzer.
Syntax tree is processed by Semantic analyzer.
Intermediate code such as 3 –address code is used for code generation process.

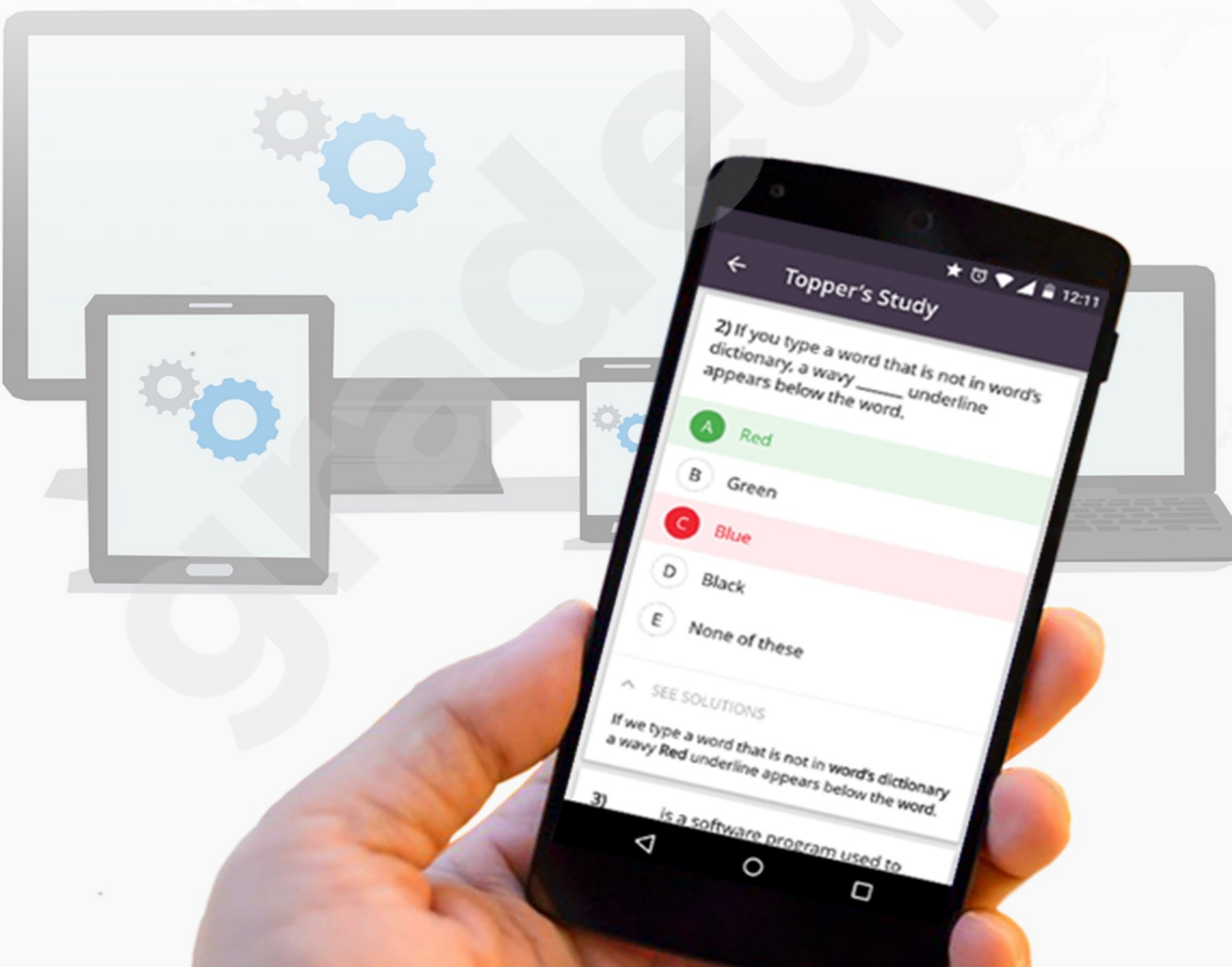
19. Ans. B.
$7 \downarrow 3 \uparrow 4 \uparrow 3 \downarrow 2$
$\Rightarrow 7 \downarrow 3 \uparrow (4 \uparrow 3) \downarrow 2$ as $\uparrow$ is right associative
$\Rightarrow 7 \downarrow (3 \uparrow (4 \uparrow 3)) \downarrow 2$
$\Rightarrow (7 \downarrow (3 \uparrow (4 \uparrow 3))) \downarrow 2$ as $\downarrow$ is left associative

20. Ans. A.
First(X) - It is the set of terminals that begin the strings derivable from X.
Follow(X) - It is the set of terminals that can appear immediately to the right of X in some sentential form.
Now in the above question,
FIRST(S) = { a, b, epsilon}
FIRST(A) = FIRST(S) = { a, b, epsilon}
FIRST(B) = FIRST(S) = { a, b, epsilon}
FOLLOW (A) = { b , a }
FOLLOW (S) = { $ } U FOLLOW (A) = { b , a , $ }
FOLLOW (B) = FOLLOW (S) = { b ,a , $ }
epsilon corresponds to empty string.

gradeup

Operating System

Questions & Solutions



Operating Systems

Direction (1-2): A computer uses 46-bit virtual address, 32-bit physical address, and a three-level paged page table organization. The page table base register stores the base address of the first-level table (T_1), which occupies exactly one page. Each entry of T_1 stores the base address of a page of the second-level table (T_2). Each entry of T_2stores the base address of a page of the third-level table (T_3). Each entry of T_3 stores a page table entry (PTE). The PTE is 32 bits in size. The processor used in the computer has a 1 MB 16-way set associative virtually indexed physically tagged cache. The cache block size is 64 bytes.

1. What is the size of a page in KB in this computer?

A. 2 B. 4 C. 8 D. 16

2. A processor uses 36 bit physical addresses and 32 bit virtual addresses, with a page frame size of 4 Kbytes. Each page table entry is of size 4 bytes. A three level page table is used for virtual-to physical address translation, where the virtual address is used as follows

* bits 30-31 are used to index into the first level page table,
* bits 21-29 are used to index into the second level page table,
* bits 12-20 are used to index into the third level page table, and
* bits 0-11 are used as offset within the page.

The number of bits required for addressing the next level page table (or page frame) in the page table entry of the first, second and third level page tables are respectively

A. 20, 20 and 20 B. 24, 24 and 24 C. 24, 24 and 20 D. 25, 25 and 24

3. A computer system implements 8 kilobyte pages and a 32-bit physical address space. Each page table entry contains a valid bit, a dirty bit, three permission bits, and the translation. If the maximum size of the page table of a process is 24 megabytes, the length of the virtual address supported by the system is___________ bits.

A. 36 B. 56 C. 30 D. 42

4. Consider a computer system with 40-bit virtual addressing and page size of sixteen kilobytes. If the computer system has a one-level page table per process and each page table entry requires 48 bits, then the size of the per-process page table is __________ megabytes.

A. 384 B. 423 C. 512 D. 643

5. Suppose a process has only the following pages in its virtual address space: two contiguous code pages starting at virtual address 0×00000000, two contiguous data pages starting at virtual address 0×00400000, and a stack page starting at virtual address 0×FFFFF000. The amount of memory required for storing the page tables of this process is

A. 8 KB B. 12 KB C. 16 KB D. 20 KB

Direction (6-7): Barrier is a synchronization construct where a set of processes synchronizes globally i.e. each process in the set arrives at the barrier and waits for all others to arrive and then all processes leave the barrier. Let the number of processes in the set be three and S be a binary semaphore with the usual P and V functions. Consider the following C implementation of a barrier with line numbers shown on left.

```
void barrier (void) {
1: P(S);
2: process_arrived++;
3. V(S);
4: while (process_arrived !=3);
5: P(S);
```

```
6: process_left++;
7: if (process_left==3) {
8: process_arrived = 0;
9: process_left = 0;
10: }
11: V(S);
}
```

The variables process_arrived and process_left are shared among all processes and are initialized to zero. In a concurrent program all the three processes call the barrier function when they need to synchronize globally.

6. The above implementation of barrier is incorrect. Which one of the following is true?

A. The barrier implementation is wrong due to the use of binary semaphore S

B. The barrier implementation may lead to a deadlock if two barrier in invocations are used in immediate succession.

C. Lines 6 to 10 need not be inside a critical section

D. The barrier implementation is correct if there are only two processes instead of three.

7. Which one of the following rectifies the problem in the implementation?

A. Lines 6 to 10 are simply replaced by process_arrived—

B. At the beginning of the barrier the first process to enter the barrier waits until process_arrived becomes zero before proceeding to execute P(S).

C. Context switch is disabled at the beginning of the barrier and re-enabled at the end.

D. The variable process_left is made private instead of shared

8. Fetch_And_Add(X, i) is an atomic Read-Modify-Write instruction that reads the value of memory location X, increments it by the value i, and returns the old value of X. It is used in the pseudocode shown below to implement a busy-wait lock. L is an unsigned integer shared variable initialized to 0. The value of 0 corresponds to lock being available, while any non-zero value corresponds to the lock being not available.

```
AcquireLock (L) {
while (Fetch_And_Add (L,1))
L = 1;
}
ReleaseLock (L) {
L = 0;
}
```

This implementation

A. fails as L can overflow

B. fails as L can take on a non-zero value when the lock is actually available

C. works correctly but may starve some processes

D. works correctly without starvation

9. A certain computation generates two arrays a and b such that a[i]=f(i)for 0 ≤ i < n and b[i] = g (a[i]) for 0 ≤ i < n. Suppose this computation is decomposed into two concurrent processes X and Y such that X computes the array a and Y computes the array b. The processes employ two binary semaphores R and S, both initialized to zero. The array a is shared by the two processes. The structures of the processes are shown below.

Operating Systems

```
Process X:                      Process Y:
private i;                      private i;
for (i=0; i<n; i++) {           for (i=0; i<n; i++) {
   a[i] = f(i);                    EntryY(R, S);
   ExitX(R, S);                    b[i] = g(a[i]);
}                               }
```

Which one of the following represents the CORRECT implementations of Exit X and Entry Y?

A.
```
ExitX(R, S) {
   P(R);
   V(S);
}
EntryY(R, S) {
   P(S);
   V(R);
}
```

B.
```
ExitX(R, S) {
   V(R);
   V(S);
}
EntryY(R, S) {
   P(R);
   P(S);
}
```

C.
```
ExitX(R, S) {
   P(S);
   V(R);
}
EntryY(R, S) {
   V(S);
   P(R);
}
```

D.
```
ExitX(R, S) {
   V(R);
   P(S);
}
EntryY(R, S) {
   V(S);
   P(R);
}
```

10. A multithreaded program P executes with x number of threads and uses y number of locks for ensuring mutual exclusion while operating on shared memory locations. All locks in the program are non-re-entrant, i.e., if a thread holds a lock l, then it cannot re-acquire lock l without releasing it. If a thread is unable to acquire a lock, it blocks until the lock becomes available. The minimum value of x and the minimum value of y together for which execution of P can result in a deadlock are:

A. x = 1, y = 2 B. x = 2, y = 1 C. x = 2, y = 2 D. x = 1, y = 1

11. Consider the following proposed solution for the critical section problem. There are n processes: $P_0 \ldots P_{n-1}$. In the code, function pmax returns an integer not smaller than any of its arguments. For all i, t[i] is initialized to zero.

```
Code for Pi:
do {
     c[i]=1; t[i] = pmax(t[0],...,t[n-1])+1;
     c[i]=0;
     for every j = i in {0,. . .,n-1} {
       while (c[j]);
       while (t[j] != 0 && t[j]<=t[i]);
     }
     Critical Section;
     t[i]=0;
     Remainder Section;
}    while (true);
```

Which one of the following is TRUE about the above solution?

A. At most one process can be in the critical section at any time
B. The bounded wait condition is satisfied
C. The progress condition is satisfied
D. It cannot cause a deadlock

Operating Systems

12. Three concurrent processes X, Y, and Z execute three different code segments that access and update certain shared variables. Process X executes the P operation (i.e., wait) on semaphores a, b and c; process Y executes the P operation on semaphores b, c and d; process Z executes the P operation on semaphores c, d, and a before entering the respective code segments. After completing the execution of its code segment, each process invokes the V operation (i.e., signal) on its three semaphores. All semaphores are binary semaphores initialized to one. Which one of the following represents a deadlock-free order of invoking the P operations by the processes?

A. X: P(a)P(b)P(c) Y: P(b)P(c)P(d) Z: P(c)P(d)P(a)
B. X: P(b)P(a)P(c) Y: P(b)P(c)P(d) Z: P(a)P(c)P(d)
C. X: P(b)P(a)P(c) Y: P(c)P(b)P(d) Z: P(a)P(c)P(d)
D. X: P(a)P(b)P(c) Y: P(c)P(b)P(d) Z: P(c)P(d)P(a)

13. A system has n resources $R_0,\ldots, R_{n-1}$, and k processes $P_0,\ldots.\ P_{k-1}$. The implementation of the resource request logic of each process P_i. is as follows:

```
if ( i% 2 = = 0) {
if (i < n) request Ri;
if (i + 2 < n) request Ri+2;
}
else {
if (i < n) request Rn–i ;
if (i + 2 < n)request Rn–i–2;
}
```

In which one of the following situations is a deadlock possible?

A. n = 40, k = 26
B. n = 21, k = 12
C. n = 20, k = 10
D. n = 41, k = 19

14. Consider the following snapshot of a system running n processes. Process i is holding $x_i$instances of a resource R, 1 ≤ i ≤ n. currently, all instances of R are occupied. Further, for all i, process i has placed a request for an additional y instances while holding the $x_i$instances it already has. There are exactly two processes p and q such that 0. y_p=y_q=Which one of the following can serve as a necessary condition to guarantee that the system is not approaching a deadlock?

A. $\min(x_p, x_q) < \max_{k \neq p,q} y_k$
B. $x_p + x_q \geq \min_{k \neq p,q} y_k$
C. $\max(x_p, x_q) > 1$
D. $\min(x_p, x_q) > 1$

15. Consider a uniprocessor system executing three tasks T_1, T_2 and T_3, each of which is composed of an infinite sequence of jobs (or instances) which arrive periodically at intervals of 3, 7 and 20 milliseconds, respectively. The priority of each task is the inverse of its period, and the available tasks are scheduled in order of priority, with the highest priority task scheduled first. Each instance of T_1, T_2 and T_3 requires an execution time of 1, 2 and 4 milliseconds, respectively. Given that all tasksinitially arrive at the beginning of the 1^{st}millisecond and task preemptions are allowed, the first instance of T_3 completes its execution at the end of ___________ milliseconds.

A. 5
B. 10
C. 12
D. 15

16. Three processes A, B and C each execute a loop of 100 iterations. In each iteration of the loop, a process performs a single computation that requires t_c CPU milliseconds and then initiates a single I/O operation that lasts for t_{io} milliseconds. It is assumed that the computer where the processes execute has sufficient number of I/O devices and the OS of the computer assigns different I/O devices to each process. Also, the scheduling overhead of the OS is negligible. The processes have the following characteristics:

Operating Systems

Process id	t_c	t_{io}
A	100ms	500ms
B	350ms	500ms
C	200ms	500ms

The processes A, B, and C are started at times 0, 5 and 10 milliseconds respectively, in a pure time sharing system (round robin scheduling) that uses a time slice of 50 milliseconds. The time in milliseconds at which process C would complete its first I/O operation is ____.

A. 1000
B. 2000
C. 3000
D. 4000

17. Consider three processes, all arriving at time zero, with total execution time of 10, 20 and 30 units, respectively. Each process spends the first 20% of execution time doing I/O, the next 70% of time doing computation, and the last 10% of time doing I/O again. The operating system uses a shortest remaining compute time first scheduling algorithm and schedules a new process either when the running process gets blocked on I/O or when the running process finishes its compute burst. Assume that all I/O operations can be overlapped as much as possible. For what percentage of time does the CPU remain idle?

A. 0%
B. 10.6%
C. 30.0%
D. 89.4%

18. The P and V operations on counting semaphores, where s is a counting semaphore, are defined as follows:

P(s): s = s - 1;
if s < 0 then wait;
V(s): s = s + 1;
if s <= 0 then wakeup a process waiting on s;

Assume that P_b and V_b, the wait and signal operations on bi nary semaphores are provided. Two binary semaphores X_b and Y_b are used to implement the semaphore operations P(s) and V(s) as follows:

P(S) :
P_b (X_b) ;
s = s - 1;
if (s < 0) {
V_b (X_b);
P_b (Y_b);
}
else V_b (X_b);
V(s):
P_b(X_b);
s = s + 1;
if (s <= 0) V_b(Y_b);
V_b (X_b);

The initial values of X_b and Y_b are respectively

A. 0 and 0
B. 0 and 1
C. 1 and 0
D. 1 and 1

Operating Systems

19. Consider a disk queue with requests for I/O to blocks on cylinders 47, 38, 121, 191, 87, 11, 92, 10. The C-LOOK scheduling algorithm is used. The head is initially at cylinder number 63, moving towards larger cylinder numbers on its servicing pass. The cylinders are numbered from 0 to 199. The total head movement (in number of cylinders) incurred while servicing these requests is __________.

A. 346   B. 423

C. 545   D. 623

20. Two processes X and Y need to access a critical section. Consider the following synchronization construct used by both the processes

Process X

```
/* other code for process X */
while(true)
{
   varP = true;
   while(varQ == true)
       {
       /* Critical  Section  */
              varP =  false;
        }
}
/* other code for process X */
```

Process Y

```
/* other code for process Y */
while(true)
{
   varQ = true;
   while(varP == true)
       {
       /* Critical  Section  */
              varQ =  false;
        }
}
/* other code for process Y */
```

Here, var P and var Q are shared variables and both are initialized to false. Which one of the following statements is true?

A. The proposed solution prevents deadlock but fails to guarantee mutual exclusion

B. The proposed solution guarantees mutual exclusion but fails to prevent deadlock

C. The proposed solution guarantees mutual exclusion and prevents deadlock

D. The proposed solution fails to prevent deadlock and fails to guarantee mutual exclusion

ANSWERS

1. Ans. C.

Let the page size be 2^x Bytes.

Then, the page offset = X bits

46 - x	x

Now, we are using 3-level paging. First level page table is contained in one page. Each page table entry is 32-bit.

The size of T_3 is $= \frac{2^{46} * 2^2}{2^x} = 2^{46+2-x} [\because PTE = 32\ bit = 4B = 2^2 B]$

The size of

$T_2\ is = \frac{2^{46+2x} * 2^2}{2^x} = 2^{46+4-x}$

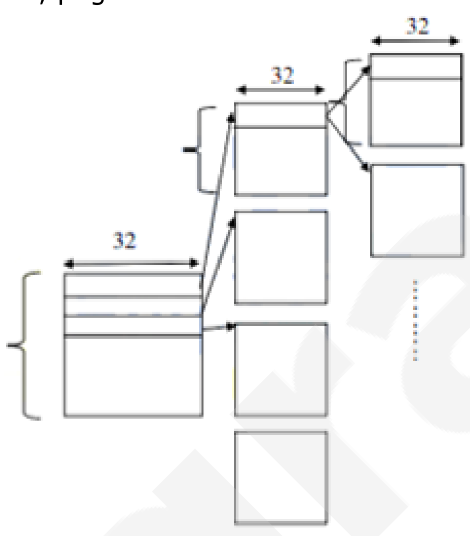
The size of

$T_1\ is = \frac{2^{46+4.2x} * 2^2}{2^x} = 2^{46+6-3x} = 2^x$

[$\because$ T_1 occupies exactly one page]

$\therefore 46 + 6 - 3x = x \Rightarrow x = 13$

So, page size = 2^{13} B = 2^3 kB = 8kB



2. Ans. D.

LAS size = 32 bits

PAS size = 36 bits

Physical memory size = 2^{36} bytes

Page frame size = 4K bytes = 2^{12} bytes

No. of bits required to access physical memory frame = 36 - 12 = 24

So in third level of page table, 24 bits are required to access an entry.

9 bits of virtual address are used to access second level page table entry and size of pages in second level is 4 bytes.

- Second level page table is $(2^9)*4 = 2^{11}$ bytes. It means there are $(2^{36})/(2^{11}) = 2^{25}$ possible locations to store this page table.

So second page table requires 25 bits to address it. Similarly, the third page table needs 25 bits to address it

ANSWERS

3. Ans. A.
Max size of virtual address can be calculated by calculating maximum number of page table entries.
Maximum Number of page table entries can be calculated using given maximum page table size and size of a page table entry.
Given maximum page table size = 24 MB
Let us calculate size of a page table entry.
A page table entry has following number of bits.
1 (valid bit) +
1 (dirty bit) +
3 (permission bits) +
x bits to store physical address space of a page.
Value of x = (Total bits in physical address) -
(Total bits for addressing within a page)
Since size of a page is 8 kilobytes, total bits needed within
a page is 13.
So value of x = 32 - 13 = 19
Putting value of x, we get size of a page table entry =
1 + 1 + 3 + 19 = 24bits.
Number of page table entries
= (Page Table Size) / (An entry size)
= (24 megabytes / 24 bits)
= 2^{23}
Virtual address Size
= (Number of page table entries) * (Page Size)
= 2^{23} * 8 kilobits
= 2^{36}
Therefore, length of virtual address space = 36

4. Ans. A.
Given $LA = 40\ bit = LAS = 2^{40}$
Page size = 16 KB
Page table Entry size (e) = 48 bits (or) 6 bytes
Page table size = ?
Size of the page table $= n \times e$
∴ No. of pages (n) $= \frac{LAS}{PS} = \frac{2^{40}}{2^{14}} = 2^{26} = 64M$
∴ Page table size $= 64 \times 6B = 384MB$

5. Ans. C.
First level page table is addressed using 10 bits and hence contains 2^{10} entries. Each entry is 4 bytes and hence this table requires 4 KB. Now, the process uses only 3 unique entries from this 1024 possible entries (two code pages starting from 0x00000000 and two data pages starting from 0x00400000 have same first 10 bits). Hence, there are only 3 second level page tables. Each of these second level page tables are also addressed using 10 bits and hence of size 4 KB. So,
total page table size of the process
= 4 KB + 3 * 4 KB
= 16 KB

ANSWERS

6. Ans. B.

It is mentioned that there are 3 processes.

Let it be P1, P2 & P3. Every process arrives at the barrier that completes its first section of the code. Hence process_arrived = 3. Now assume that P1 continues its execution and leaves the barrier and makes process_left=1. Now assume that P1 again invokes the barrier function immediately. At this stage, process_arrived =4. P1 now waits. Soon P2 leaves the barrier, which makes process_left=2. Now P3 leaves the barrier which makes process_left=3. "If" condition is true and now process_arrived and process_left is reset to 0.

We know that P1 is waiting. At this stage assume that P2 invokes the barrier function, hence process_arrived=1 and it waits. P3 invokes the barrier function, hence process_arrived=2. All process have reached the barrier but since the process_arrived=2, all processes keeps on waiting. Hence option (B) is correct.

7. Ans. B.

To prevent variable process_arrived becoming greater than 3, Step '2' should not be executed when the process enters the barrier second time till other two processes have not completed their 7th step.

Hence, when variable process_arrived becomes zero and variable process_left also becomes zero then the problem of deadlock will be resolved.

8. Ans. B.

1) Acquire lock (L) {
2) While (Fetch_And_Add (L,1))
3) L = 1
}
4) Release Lock (L) {
5) L = 0;
6) }

Let P and Q be two concurrent processes in the system currently executing as follows

P executes 1, 2, 3 then Q executes 1 and 2 then P executes 4, 5, 6 then L = 0 now

Q executes 3 by which L will be set to 1 and thereafter no process can set

L to zero, by which all the processes could starve.

9. Ans. C.

Option (A) : Suppose process X executes Exit X then it will wait for R, and then process Y executes Entry Y then it will wait for S. Sine initially both binary semaphore are 0, no one will increment it and both process will stuck in dead lock.

Hence option (A) is wrong.

Option (B): Here if process X executes for n times repeadetly it will set both semaphores to 1 (since only two values are possible) and after that process Y executes. First time it passes the Entry Y and make both semaphores to 0. And on second time it finds both semaphores to 0 and cannot pass the Entry Y barrier. Hence it will stuck.

So option (B) is wrong.

Option (D): Suppose first process X executes it sets R = 1 and then waits for S. Now after that process Y executes. It first sets S = 1 and then decrement R = 0. It comes again and then again sets S = 1 (i.e. it overlaps the value of S) and then wait for R.

Clearly here we lost one iteration of process X due to overlapping of value of S. and after n-1 iteration process X will stuck.

So option (d) is wrong.

Option (C): Here take any sequence of operation of process X and process Y, first process X will wait for S which is increment by process Y and then process Y waits for R which is incremented by process X.

There is no sequence of operation in which the value of R or S overlaps.

Hence both process executes one after another.

So option (C) is correct.

ANSWERS

10. Ans. C.

As per given question, there 'x' number of threads and 'y' number of locks for ensuring mutual exclusion while operating on shared memory locations

Option A: x = 1; y = 2

Means that 1 thread and 2 locks clearly showing that no deadlock situation

Option B: x = ; y = 1

Means that 2 threads and 1 lock → No deadlock situation

After usage of lock by 1 thread, it can release that lock and then 2nd thread can be used that lock. So no deadlock

Option C: x = 2; y = 2

Means that 2 threads and 2 locks → Deadlock can arise

Both threads can hold 1 lock and can wait for release of another lock

Option D: x = 1; y = 1

Means that 1 thread and 1 lock → No deadlock situation

Hence Option (C) is correct.

11. Ans. A.

It can guarantee that at most one process can be in critical section at any time. But other conditions will fail in some cases.

12. Ans. B.

A. X Y Z
P(a) P(b) P(c)
P(b) P(c) P(d)
P(c) P(d) P(a)

Wrong answer because:

Suppose X execute first & executed P(a) & P(b) then process switches to Z, then Z executed P(c), P(d) & wait for P(a) then again process switches to X & X then wait for P(c).

So, X is waiting for C which is occupied by Z & z is waiting for X so no one can execute & deadlock occur.

B. X Y Z
P(b) P(b) P(a)
P(a) P(c) P(c)
P(c) P(d) P(d)

Correct because all process does not have reverse call for wait on any variable.

C. X Y Z
P(b) P(c) P(a)
P(a) P(b) P(c)
P(c) P(d) P(d)

wrong answer because sequence of P(b) & P(c) are reverse or opposite in X & Y respectively so deadlock may occur.

D. X Y Z
P(a) P(c) P(c)
P(b) P(b) P(d)
P(c) P(d) P(a)

Wrong answer because sequence of P(c) & P(a) are opposite in Z & X. So deadlock may occur.

13. Ans. B.

if n = 21, k = 12

(if (i% 2 = = 0) {if (i < n) Request Ri;

if (i + 2 < n) Request R,

else (if(i<n) Request R_{n-i}, if (i + 2 < n) Request (R_{n-i-2i})

We seen that if value of k = 11, 12

ANSWERS

Now first k = 11 means p11-1 = p_{10} = pi $\Rightarrow$ i = 0
then according to logic
initially it goes if block because (i%2 = 0)
(10%2 = = 0)
then it Request R_{10}, R_{10+2} (R12)
$\Rightarrow R_{10}, R_{12}$ Resources
If now next k = 12 then i = 11
Now it goes in else block and Request R_{n-i}, R_{n-i-2}
$\Rightarrow R_{21-11}, R_{21-11-2}$
$\Rightarrow R_{10}, R_{8}$
then it also Request R_{10} then both k = 11, k = 12 request R_{10},
It is problem of dead lock.

14. Ans. B.
Since both p and q don't need additional resources, they both can finish and release Xp + Xq resources without asking for any additional resource. If the resources released by p and q are sufficient for another process waiting for Y_k resources, then system is not approaching deadlock.

15. Ans. C.
Periods of T1, T2 and T3 are 3ms, 7ms and 20ms
Since priority is inverse of period, T1 is the highest priority task, then T2 and finally T3
Every instance of T1 requires 1ms that of T2 requires 2ms and that of T3 requires 4ms
Initially all T1, T2 and T3 are ready to get processor,
T1 is preferred
Second instances of T1, T2, and T3 shall arrive at 3, 7, and 20 respectively.
Third instance of T1, T2 and T3 shall arrive at 6, 14, and 49 respectively.

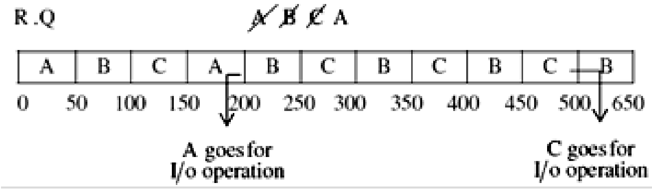
Time-Interval	Tasks
0-1	T1
1-2	T2
2-3	T2
3-4	T1 [Second Instance of T1 arrives]
4-5	T3
5-6	T3
6-7	T1 [Third Instance of T1 arrives] [Therefore T3 is preempted]
7-8	T2 [Second instance of T2 arrives]
8-9	T2
9-10	T1 [Fourth Instance of T1 arrives]
10-11	T3
11-12	T3 [First Instance of T3 completed]

16. Ans. A.

Process id	t_c	t_{io}	A.T	TQ = 50 ms
A	100 ms	500 ms	0 ms	
B	350 ms	500 ms	5 ms	
C	200 ms	500 ms	10 ms	

ANSWERS

The Gantt chart for Round robin algorithm for the first iteration execution for each of the 3 processes is as follows:



After finishing t_c CPU ms at time 500ms, C goes for I/O operation, that needs 500ms more, so the time at which process C would complete its first I/O operations is 500+500 = 1000ms

17. Ans. B.

Let us consider three processes as P_0, P_1 and P_2 with execution time as 10, 20 and 30 units respectively. The P_0, P_1 and P_2 will spend the time as below;

Process	Time on I/O	Computation	Time on I/O
P_0	2	7	1
P_1	4	14	2
P_2	6	21	3

These three processes will be executed as follows:

Time frame	Event
0-2	Idle
2-9	P_0
9-23	P_1
23-44	P_2
44-47	Idle

Hence the percentage of idle time = $\frac{5}{47} \times 100$ = 10.6 %

18. Ans. C.

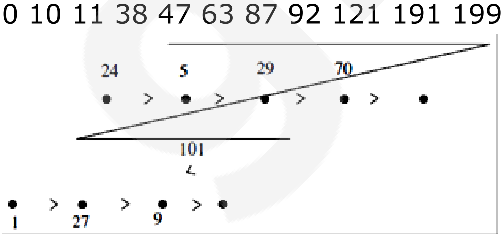
Initial values of xb and yb are 1 and 0

Hence (C) is the correct answer.

19. Ans. A.

C-Look disc Scheduling

0 10 11 38 47 63 87 92 121 191 199



∴ Total Head movements

$= 24 + 5 + 29 + 70 + 181 + 1 + 27 + 9 = 346$

20. Ans. A.

When both processes try to enter critical section simultaneously, both are allowed to do so since both shared variables var P and var Q are true. So, clearly there is NO mutual exclusion. Also, deadlock is prevented because mutual exclusion is one of the four conditions to be satisfied for deadlock to happen. Hence, answer is A.

gradeup

Discrete Maths

Questions & Solutions

Discrete Mathematics

1. If the ordinary generating function of a sequence $\{a_n\}_{n=0}^{\infty}$ is $\frac{1+x}{(1-z)^3}$, then $a_3 - a_0$ is equal to _____.

A. 13 B. 15 C. 21 D. 27

2. Suppose that the robot is not allowed to traverse the line segment from (4,4) to(5,4). With this constraint, how many distinct paths are there for the robot to reach (10,10) starting from (0,0)?

A. 2^9

B. 2^{19}

C. $\binom{8}{4}\times\binom{11}{5}$

D. $\binom{20}{10}-\binom{8}{4}\times\binom{11}{5}$

Direction (3-4): Suppose that a robot is placed on the Cartesian plane. At each step it is allowed to move either one unit up or one unit right, i.e., if it is at (i, j) then it can move to either (i + 1, j) or (i, j + 1).

3. How many distinct paths are there for the robot to reach the point (10,10)starting from the initial position (0,0)?

A. $\binom{20}{10}$ B. 2^{20} C. 2^{10} D. None of the above

4. A pennant is a sequence of numbers, each number being 1 or 2. An n-pennant is a sequence of numbers with sum equal to n. For example, (1,1,2) is a 4-pennant. The set of all possible 1-pennants is {(1)}, the set of all possible 2-pennants is {(2), (1,1)} and the set of all 3-pennants is {(2,1), (1,1,1), (1,2)}. Note that the pennant (1,2) is not the same as the pennant (2,1). The number of 10-pennants is __________.

A. 87 B. 88 C. 89 D. 90

5. Consider the first-order logic sentence

F: ∀ x (∃ y R(x,y)).

Assuming non-empty logical domains, which of the sentences below are implied by F?

I. ∃y (∃x R(x,y))

II. ∃y (∀x R(x,y))

III. ∀y (∃x R(x,y))

IV. ~∃x (∀y R(x,y))

A. IV only B. I and IV only C. II only D. II and III only

6. Which one of the following well-formed formulae in predicate calculus is NOT valid?

A. (∀x p(x) ⇒ ∀x q(x)) ⇒ (∃x ¬ p(x) ∨ ∀x q(x))

B. (∃x p(x) ∨ ∃x q(x)) ⇒ ∃x (p(x) ∨ q(x))

C. ∃x (p(x) ∧ q(x)) ⇒ (∃x p(x) ∧ ∃x q(x))

D. ∀x (p(x) ∨ q(x)) ⇒ (∀x p(x) ∨ ∀x q(x))

7. Which one of the following options is **CORRECT** given three positive integers *x, y* and *Z*, and a predicate

$$P(x) = \neg(x=1) \wedge \forall y\left(\exists z(x = y * z) \Rightarrow (y = x) \vee (y=1)\right)$$

A. *P* (*x*) being true means that *x* is a prime number

B. *P* (*x*) being true means that *x* is a number other than 1

C. *P*(*x*) is always true irrespective of the value of *x*

D. *P*(*x*) being true means that *x* has exactly two factors other than 1 and *x*

Discrete Mathematics

8. A function $f : N^+ \rightarrow N^+$, defined on the set of positive integers N^+, satisfies the following properties

$f(n) = f(n/2)$ if n is even

$f(n) = f(n+5)$ if n is odd

Let $R = \{i \mid \exists j : f(j) = i\}$ be the set of distinct values that f takes. The maximum possible size of R is ____________.

A. 2 B. 3 C. 4 D. 5

9. There are two elements x, y in a group (G,*) such that every element in the group can be written as a product of some number of x's and y's in some order. It is known that

$x * x = y * y = x * y * x * y = y * x * y * x = e$

where e is the identity element. The maximum number of elements in such a group is __________.

A. 2 B. 3 C. 4 D. 5

10. A binary relation R on N × N is defined as follows: (a, b)R(c, d) if a ≤ c or b ≤ d.
Consider the following propositions:
P: R is reflexive
Q: R is transitive
Which one of the following statements is TRUE?
A. Both P and Q are true
B. P is true and Q is false
C. P is false and Q is true
D. Both P and Q are false

11. Consider the following relations:
R1 (a,b) if (a+b) is even over the set of integers
R2 (a,b) if (a+b) is odd over the set of integers
R3 (a,b) if a.b> 0 over the set of non-zero rational numbers
R4 (a,b) if |a – b| ≤2 over the set of natural numbers
Which of the following statements is correct?
A. R1 and R2 are equivalence relations, R3 and R4 are not
B. R1 and R3 are equivalence relations, R2 and R4 are not
C. R1 and R4 are equivalence relations, R2 and R3 are not
D. R1, R2, R3 and R4 are all equivalence relations

12. A relation R is defined on ordered pairs of integers as follows:
(x, y) R(u, v) if x < u and y > v. Then R is:
A. Neither a Partial Order nor an Equivalence Relation
B. A Partial Order but not a Total Order
C. A Total Order
D. An Equivalence Relation

13. How many graph on n labeled vertices exist which have at least $\frac{n-3n}{2}$ edges.

A. ${}^{(n^2-n)/2}C_{(n^2-3n)/2}$

B. $\sum_{k=0}^{(n^2-3n)/2} {}^{(n^2-n)}C_k$

C. ${}^{(n^2-n)/2}C_n$

D. $\sum_{k=0}^{n} {}^{(n^2-n)/2}C_k$

Discrete Mathematics

14. The maximum degree of a vertex in G is:

A. $\binom{n/2}{2}2^{n/2}$ B. 2^{n-2} C. $2^{n-3} \times 3$ D. 2^{n-1}

Direction (15-17): The 2n vertices of a graph G corresponds to all subsets of a set of size n, for n ≥6. Two vertices of G are adjacent if and only if the corresponding sets intersect in exactly two elements.

15. The number of vertices of degree zero in G is:

A. 1 B. n C. n +1 D. 2^n

16. Consider an undirected graph G where self-loops are not allowed. The vertex set of G is {(i, j): 1 ≤ i ≤ 12, 1 ≤ j ≤ 12}. There is an edge between (a, b) and (c, d) if |a - c| ≤ 1 and |b - d| ≤ 1.The number of edges in this graph is __________.

A. 500 B. 502 C. 506 D. 510

17. If G is a forest with n vertices and k connected components, how many edges does G have?

A. [n/k] B. [k/n] C. n – k D. n - k + 1

18. The line graph *L*(*G*) of a simple graph *G* is defined as follows:

* There is exactly one vertex *v*(*e*) in *L*(*G*) for each edge e in G.

* For any two edges e and e' in G, L(G) has an edge between v(e) and v(e'), if and only if e and e' are incident with the same vertex in G.

Which of the following statements is/are TRUE?

(P) The line graph of a cycle is a cycle.

(Q) The line graph of a clique is a clique.

(R) The line graph of a planar graph is planar.

(S) The line graph of a tree is a tree.

A. P only B. P and R only C. R only D. P, Q and S only

19. Let G = (V,E) be a graph. Define ξ(G) = Σd id x d, where id is the number of vertices of degree d in G. If S and T are two different trees with ξ(S) = ξ(T),then

A. |S| = 2|T| B. |S| = |T| −1 C. |S| = |T| D. |S| = |T| + 1

20. Identify the correct translation into logical notation of the following assertion. Some boys in the class are taller than all the girls

Note: taller (x, y) is true if x is taller than y.

A. (∃x) (boy(x) → (∀y) (girl(y) ∧ taller(x,y))

B. (∃x) (boy(x) ∧ (∀y) (girl(y) ∧ taller(x,y))

C. (∃x) (boy(x) → (∀y) (girl(y) → taller(x,y))

D. (∃x) (boy(x) ∧ (∀y) (girl(y) → taller(x,y))

ANSWERS

1. Ans. B.

Let $\frac{1+Z}{(1-Z)^3} = a_0 + a_1 Z + a_2 Z^2 + a_3 Z^3 + \ldots\ldots$------(1)

$\frac{1+Z}{(1-Z)^3} = (1+Z)(1-Z)^{-3}$

$= (1+Z)(1+3Z + 6Z^2 + 10Z^3 + \ldots\ldots\ldots\infty)$

Using binomial theorem

$= 1 + 4Z + 9Z^2 + 16Z^3 + \ldots\ldots \infty$………(2)

From (1) and (2), $a_0 = 1$ and $a_3 = 16$

$\therefore a_3 - a_0 = 15$

2. Ans. D.

Since we are not allowed to traverse from (4: 4) to (5 : 4).

So we will subtract all those paths which were passing through (4:4) to (5:4).

To count number of paths passing through (4:4) to (5:4): we find number of paths from (0:0) to (4:4): and then from (5:4) to (10:10).

From (0:0) to (4:4): number of paths = $^8C_4$.

From (5:4) to (10:10): number of paths = $^{11}C_5$.

So total number of paths required: $^{20}C_{10}$ - $^8C_4 \times {}^{11}C_5$

So option (D) is correct.

3. Ans. A.

We see that if the robot makes 20 moves it must reach the final position. At each move, robot can move either 1 unit right or 1 unit up, and there will be 20 such moves required to reach (10:10) from (0:0). So we have to divide these 20 moves, numbered from 1 to 20: into 2 groups: right group and up group. Right group contains those moves in which we move right, and up group contains those moves in which we move up. Each group contains 10 elements each. So basically, we have to hide 20 things into 2 groups of 10 10 things each, which can be done in $\frac{20!}{10! \times 10!}$ = $^{20}C_{10}$ ways.

So option (A) is correct.

4. Ans. C.

No twos = 1111111111 = 1 pennant

Single twos = 211111111 = (9!/(8!1!)) =9 pennants

Two twos: 22111111 = (8!/(6!2!)) = 28 pennants

Three twos: 2221111 = (7!/(3!4!)) = 35 pennants

Four twos: 222211= (6!/(4!2!)) = 15 pennants

Five twos: 22222=1 pennant i.e. Total =89 pennants.

5. Ans. B.

$\forall x(\exists y R(x,y)) \Rightarrow \exists y \exists x R(x,y)$

$\exists y \forall x R(x,y) \Rightarrow \forall x \exists y R(x,y)$

$\forall x \exists y R(x,y) \not\Rightarrow \exists y \forall x R(x,y)$

$\neg \exists x(\forall y \neg R(x,y)) \Leftrightarrow \forall x \exists y R(x,y)$

ANSWERS

6. Ans. D.

For every x , p or q is true. It can not imply that Either for every x, p is true OR for every x, q is true. So last option is wrong.

7. Ans. A.

The given statement reads,

P(X) is true whenever x is not 1 and for every y, if there is a z such that x = y *z, then either x = y or y = 1

In other words, P(x) is true whenever x is prime.

8. Ans. A.

Given $f(n) = f\left(\frac{n}{2}\right)$ is n is even

$= f(n+5)$ if n is odd

We can observe that $f(1) = f(2) = f(3) = f(4) = f(6) = f(7)\ldots$ and $f(5) = f(10) = f(15) = \ldots\ldots$

Clearly, the range of f(x) will contain two distinct elements only.

9. Ans. C.

x * x = e, x is its own inverse

y * y = e, y is its own inverse

(x*y) * (x* y) = e, x*y is its own inverse

(y*x) * (y*x) = e, y*x is its own inverse

Also x*x*e = e*e can be rewritten as follows,

x*y*y*x = e*y*y*e = e, (Since y *y = e)

(x*y) * (y*x) = e shows that (x *y) and (y *x) are each other's inverse and we already know that (x*y) and (y*x) are inverse of its own. As per (G,*) to be group any element should have only one inverse element (unique). This implies x*y = y*x (is one element). So the elements of such group are 4 which are {x, y, e, x*y}.

10. Ans. B.

It is reflexive as every ordered pair is related to itself

(a,b) R (a,b) since $a \le a$ or $b \le b$

It is not transitive as (2,4)R(3,2) & (3,2) R(1,3) but (2,4) R (1,3)

11. Ans. B.

R1 is reflexive, R2 is symmetric, R3 is reflexive and transitive. but R2 is not transitive. For example take (1, 2) (2, 3) (1, 3), conditions are not satisfied. R4 is not transitive. For example take (1, 3) (3, 5) (1, 5), here also conditions are not satisfied. R1 and R3 are equivalence relations, R2 and R4 are not

12. Ans. A.

We know that a relation R is partial Order if it is reflexive, antisymmetric, and transitive.

Also, R is Equivalence Relation if it is reflexive, symmetric, and transitive.

R is not reflexive because (x, y)R(x, y); x $\nless$ x

So the relation R is neither a Partial order nor an Equivalence Relation.

ANSWERS

13. Ans. D.

If I straight away do this question then it should be -

$\sum_{k=\frac{n^2-3n}{2}}^{\frac{n^2-n}{2}} \binom{\frac{n^2-n}{2}}{k}$, None of the option matches. Therefore I need to modify this little bit to come up in the same format of one of the option.

(See, Option A and C are out of consideration. **A** is the number of graphs with exactly equal to $\frac{n^2-3n}{2}$ edges. And Option **C** is the number of graphs with exactly equal to n edges).

Once you realize one identity then this problem is handy to u-

$\sum_{k=r}^{n} \binom{n}{k} = \sum_{k=0}^{n-r} \binom{n}{k}$ (to prove this- see, first term of LHS is same as last term of RHS and 2nd term equate to 2nd last term of RHS and so on. And number of terms in LHS and RHS are same i.e. $n-r$)

Using this, we can see

$$\sum_{k=\frac{n^2-3n}{2}}^{\frac{n^2-n}{2}} \binom{\frac{n^2-n}{2}}{k} = \sum_{k=0}^{n} \binom{\frac{n^2-n}{2}}{k}$$

Hence Option D is matching. And it is not hard to proof.

14. Ans. C.

Let the vertex having the max degree contain k elements. Now, as per the given condition, it can have edges to all vertices having two common elements.

The number of ways in which we can select these two elements will be equal to ${}^kC_2$

So, the degree of vertex in G = ${}^kC_2 \times 2^{(n-k)}$

Maximum degree of a vertex G = ${}^kC_2 \times 2^{(n-k)} = {}^3C_2 \times 2^{(n-3)} = 3 \times 2^{(n-3)}$

15. Ans. C.

Number of vertices with degree zero = number of subsets with size

⇒ 1 = n+1, as edges are there for every vertex with two or more elements we have a vertex for all subsets of n.

16. Ans. C.

From the given data, there can be total 12*12 possible vertices. The vertices are (1, 1), (1, 2).... (1, 12) (2, 1), (2, 2),....

Number of edges is equal to number of pairs of vertices that satisfy above conditions. For example, vertex pair {(1, 1), (1, 2)} satisfy above condition.

For (1, 1), there can be an edge to (1, 2), (2, 1), (2, 2). Note that there can be self-loop as mentioned in the question. Same is count for (12, 12), (1, 12) and (12, 1)

For (1, 2), there can be an edge to (1, 1), (2, 1), (2, 2), (2, 3), (1, 3)

Same is count for (1, 3), (1, 4).... (1, 11), (12, 2), ... (12, 11)

For (2, 2), there can be an edge to (1, 1), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2), (3, 3)

Same is count for remaining vertices.

For all pairs (i, j) there can total 8 vertices connected to them if i and j are not in {1, 12}. There are total 100 vertices without a 1 or 12. So total 800 edges.

For vertices with 1, total edges = (Edges where 1 is first part) + (Edges where 1 is second part and not first part)

= (3 + 5*10 + 3) + (5*10) edges

Same is count for vertices with 12

ANSWERS

Total number of edges:

= 800 + [(3 + 5*10 + 3) + 5*10] + [(3 + 5*10 + 3) + 5*10]

= 800 + 106 + 106

= 1012

Since graph is undirected, two edges from v1 to v2 and v2 to v1 should be counted as one. So total number of undirected edges = 1012/2 = 506.

17. Ans. C.

Each component will have n/k vertices (pigeonhole principle). Hence for each component there will be (n/k) -1 edges. Since there are k components, total number of edges= k*((n/k)-1) = n-k.

18. Ans. A.

(P) Let 'C' be a cycle. In the live graph L(C):

(i) There will be an edge between two vertices corresponding to adjacent edges in the cycle.

(ii) Degree of each vertex in L(C) will be 2

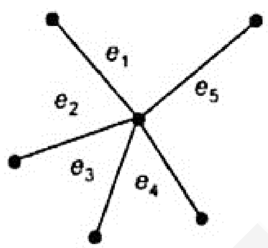
(iii) L(C) will be connected.

Hence, L(C) will also be a cycle. So, (P) is TRUE (Q) Let 'K_n' be a clique on 'n' vertices. Consider line graph L(K_n) :

- L(K_n) has n_{c2} vertices at least say e_1, e_2
- For n > 3, there will be two edges in "K_n" which do not share a vertex in "K_n".
- So, V(e_1), V(e_2) will not be adjacent in L(K_n).
- Hence, L(K_n) is not always a clique.

(Q) is FALSE

(R) Let 'G' be a star graph consider line graphical



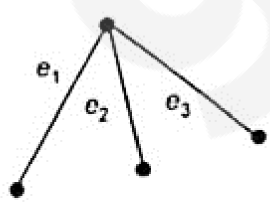
$\Rightarrow In\ L(G): V(e_1), V(e_2), V(e_3), V(e_4), V(e_5)$ Will "K5" from a clique because they all share a vertex.

A planar graph cannot have a "K5" minor. Hence L(G) is not a planar graph.

Even though G is planar

So, R is FALSE

(s) Let "T" be the following tree, and L(T) is its line graph



- In L(T) : e_1, e_2, e_3 will form a K_3 $K_3(\Delta L_e)$ i.e., a 3-cycle.
- Tree is an acyclic graph i.e. doesn't have a cycle.
- So, L(T) is not a tree.

So, S is False

$\therefore$ Answer is "P" only.

ANSWERS

19. Ans. C.

The expression $\xi(G)$ is basically sum of all degrees in a tree. For example, in the following tree, the sum is 3 + 1 + 1 + 1.

```
  a
 / | \
b  c  d
```

Now the questions is, if sum of degrees in trees are same, then what is the relationship between number of vertices present in both trees?

The answer is, $\xi(G)$ and $\xi(T)$ is same for two trees, then the trees have same number of vertices. It can be proved by induction. Let it be true for n vertices. If we add a vertex, then the new vertex (if it is not the first node) increases degree by 2, it doesn't matter where we add it. For example, try to add a new vertex say 'e' at different places in above example tee

20. Ans. D.

Now many people get confused when to use $\wedge$and when to use $\rightarrow$. This question tests exactly that. We use $\wedge$when we want to say that the both predicates in this statement are always true, no matter what the value of x is. We use $\rightarrow$when we want to say that although there is no need for left predicate to be true always, but whenever it becomes true, right predicate must also be true. D means there exist some boys x which taller than all girls y.

Hence, $(\exists x)$ $(boy(x) \wedge (\forall y)$ $(girl(y) \rightarrow taller(x,y))$ is the correct answer

Engineering Mathematics

1. Suppose that a shop has an equal number of LED bulbs of two different types. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7 and given that it is of Type 2 is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100 hours is ____.

A. 0.11 B. 0.23 C. 0.55 D. 0.62

2. If two fair coins are flipped and at least one of the outcomes is known to be a head, what is the probability that both outcomes are heads?

A. 1/3 B. 1/4 C. 1/2 D. 2/3

3. Suppose X_i for i=1,2,3 are independent and identically distributed random variables whose probability mass functions are $Pr[X_i=0]= Pr[X_i = 1]=1/2$ for i=1,2,3. Define another random variable $Y = X_1X_2 \oplus X_3$, where $\oplus$ denotes XOR. Then $Pr[Y=0|X_3=0]$ = _____________.

A. 0.75 B. 0.65 C. 0.98 D. 1

4. The security system at an IT office is composed of 10 computers of which exactly four are working. To check whether the system is functional, the officials inspect four of the computers picked at random (without replacement). The system is deemed functional if at least three of the four computers inspected are working. Let the probability that the system is deemed functional be denoted by p Then 100p= ____.

A. 11.9 B. 13.1 C. 14.2 D. 15.3

5. If a random variable X has a Poisson distribution with mean 5, then the expectation $E\left[(X+2)^2\right]$ equals _____

A. 34 B. 44 C. 54 D. 64

6. Suppose you break a stick of unit length at a point chosen uniformly at random. Then the expected length of the shorter stick is _____

A. 0.5 B. 0.25 C. 0.15 D. 0.75

7. Consider a finite sequence of random values $X=[x_1,x_2,\ldots\ldots\ldots,x_n]$. Let μ_x be the mean and $_x$ be the standard deviation of X. Let another finite sequence Y of equal length be derived from this as $y_i = a^*x_i + b$, where a and b are positive constants. Let μ_y be the mean and σ_y be the standard deviation of this sequence. Which one of the following statements is **INCORRECT**?

A. Index position of mode of X in X is the same as the index position of mode of Y in Y.

B. Index position of median of X in X is the same as the index position of median of Y in Y.

C. $\mu_y = a\mu_x + b$

D. $\sigma_y = a\sigma_x + b$

8. If the following system has non-trivial solution,

px+ qy+ rz= 0

qx+ ry+ pz= 0

rx+ py+ qz= 0,

then which one of the following options is TRUE?

A. p- q+ r= 0orp= q= -r B. p+ q- r= 0orp= -q= r

C. p+ q+ r= 0orp= q= r D. p- q+ r= 0orp= -q= -r

Engineering Mathematics

9. Consider the systems, each consisting of m linear equations in n variables.
   I. If m < n, then all such systems have a solution
   II. If m > n, then none of these systems has a solution
   III. If m = n, then there exists a system which has a solution
   Which one of the following is CORRECT?
   A. I, II and III are true
   B. Only II and III are true
   C. Only III is true
   D. None of them is true

10. A function f(x) is continuous in the interval [0,2]. It is known that f(0) = f(2) = -1 and f(1) = 1. Which one of the following statements must be true?
    A. There exists a y in the interval (0,1) such that f(y) = f(y+1)
    B. For every y in the interval (0,1), f(y) = f(2-y)
    C. The maximum value of the function in the interval (0,2) is 1
    D. There exists a y in the interval (0,1) such that f(y) = -f(2-y)

11. Let f (x) be a polynomial and g(x) = f (x) be its derivative. If the degree of (f(x) + f(−x)) is 10, then the degree of (g(x) − g(−x)) is ___________
    A. 8
    B. 9
    C. 10
    D. 11

12. If $\int_0^{2\pi} |x \sin x|\, dx = k\pi,$ then the value of k is equal to _______ .
    A. 1
    B. 2
    C. 3
    D. 4

13. For any discrete random variable X, with probability mass function
    $P(X=j)=p_j, p_j \geq 0, j \in \{0,\dots,N\}$ and $\sum_{j=0}^{N} p_j = 1,$ define the polynomial function
    $g_x(z) = \sum_{j=0}^{N} p_j z^j$
    For a certain discrete random variable Y, there exists a scalar $\beta \in [0,1]$ such that $g_Y(z) = (1-\beta+\beta z)^N$.
    The expectation of Y is
    A. $N\beta(1-\beta)$
    B. $N\beta$
    C. $N(1-\beta)$
    D. Not expressible in terms of N and β alone

14. Consider a quadratic equation $x^2 - 13x + 36 = 0$ with coefficients in a base b. The solutions of this equation in the same base b are x = 5 and x = 6. Then b = ______.
    A. 5
    B. 6
    C. 7
    D. 8

15. The probability that a k-digit number does NOT contain the digits 0,5,or 9 is
    A. 0.5^k
    B. 0.6^k
    C. 0.7^k
    D. 0.9^k

16. The number of integers between 1 and 500 (both inclusive) that are divisible by 3 or 5 or 7 is _______.
    A. 212
    B. 271
    C. 311
    D. 354

Engineering Mathematics

17. Let A be real valued square symmetric matrix of rank 2 with $\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}^2 = 50.$ Consider the following statements.

(I) One eigen value must be in [-5, 5]

(II) The eigen value with the largest magnitude must be strictly greater than 5.

Which of the above statements about eigen values of A is/are necessarily CORRECT?

A. Both (I) and (II)
B. (I) only
C. (II) only
D. Neither (I) nor (II)

18. Let C_1 …… C_n be scalars, not all zero, such that $\sum_{i=1}^{n} c_i a_i = 0$ where are column vectors in R^n. Consider the set of linear equations Ax = b

where A = $[a_1 \ldots . a_n]$ and $b = \sum_{i=1}^{n} a_i$. The set of equations has

A. a unique solution at x = j_n where j_n denotes a n-dimensional vector of all 1
B. no solution
C. infinitely many solutions
D. finitely many solutions

19. Consider the first-order logic sentence

F: ∀ x (∃ y R(x,y)).

Assuming non-empty logical domains, which of the sentences below are implied by F?

I. ∃y (∃x R(x,y))
II. ∃y (∀x R(x,y))
III. ∀y (∃x R(x,y))
IV. ~∃x (∀y R(x,y))

A. IV only
B. I and IV only
C. II only
D. II and III only

ANSWERS

1. Ans. C.
E_1 –event of selecting type-I bulb
E_2-event of selecting type-II bulb
A-Event of selecting a bulb lasts more than 100 hours
Given
$P(E_1) = 0.5, P(E_2) = 0.5$
$P(A/E_1) = 0.7, P(A/E_2) = 0.4$
Required probability,
$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$$
$$= 0.5 \times 0.7 + 0.5 \times 0.4$$
$$= 0.55$$

2. Ans. A.
Sample space = {HH, HT, TH}
Required probability = 1/3

3. Ans. A.
It is given X3 = 0. Y can only be 0 when X1 X2 is 0. X1 X2 become 0 for X1 = 1, X2 = 0, X1 = X2 = 0 and X1 = 0, X = 1 So the probability is = 0.5*0.5*3 = 0.75.
In probability and statistics, a probability distribution assigns a probability to each measurable subset of the possible outcomes of a random experiment, survey, or procedure of statistical inference. Examples are found in experiments whose sample space is non-numerical, where the distribution would be a categorical distribution; experiments whose sample space is encoded by discrete random variables, where the distribution can be specified by a probability mass function; and experiments with sample spaces encoded by continuous random variables, where the distribution can be specified by a probability density function. More complex experiments, such as those involving stochastic processes defined in continuous time, may demand the use of more general probability measures.
In applied probability, a probability distribution can be specified in a number of different ways, often chosen for mathematical convenience:
by supplying a valid probability mass function or probability density function
by supplying a valid cumulative distribution function or survival function
by supplying a valid hazard function
by supplying a valid characteristic function
by supplying a rule for constructing a new random variable from other random variables whose joint probability distribution is known.

4. Ans. A.
p=P [at least three computers are working]
= P (3 or 4 computers working)
$$= \frac{(4_{C_3}) \times (6_{C_1})}{10_{C_4}} + \frac{4_{C_4}}{10_{C_4}} = \frac{5}{42}$$
$$\Rightarrow 100\,p = 11.9$$

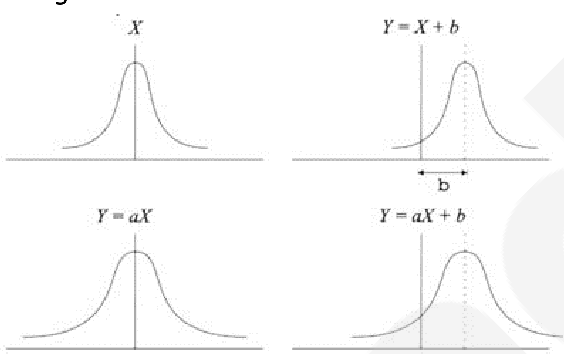
5. Ans. C.
Using Linearity of Expectation, we can write,
$E[(X+2)^2] = E[X^2] + E[4X] + E[4]$

ANSWERS

The Poisson distribution, mean and variance are same. Here Mean is given as 5. So variance should also be 5.
Also,
Variance = E[X²] – (E[X])²
5 = E[X^2] – 25.
E[X^2] = 30
Thus E[$(X+2)^2$] = 30 + 4*5 + 4 = 54.

6. Ans. B.
Suppose you break a stick in such a way that shorter stick is of length X. this division of the stick will be uniform. Hence, x has uniform distribution in the interval [0, 1/2]
Now for uniform distribution,
$\alpha = 0$ and $\beta = 0.5$
Hence, Expectation = $\beta + \alpha / 2$
therefore, E(x)= (1/2+0)/2
= 0.25

7. Ans. D.
Adding a constant like b shift the distribution while multiplying to a constant like a stretch the distribution along median.



Mode is the most frequent data of the distribution, so the index position of the mode will not change. From the above graph it is clear that index position of the median will also not change. Now for the mean

$$Y_i = aX_i + b$$
$$\sum Y_i = \sum (aX_i + b)$$
$$\sum Y_i = a(\sum X_i) + nb$$
$$(\sum Y_i)/n = a(\sum X_i)/n + b$$
$$\mu_y = a\mu_x + b$$

And for the standard deviation

$$\sigma_y = \sqrt{\frac{1}{n}\sum(\mu_y - Y_i)^2}$$
$$\sigma_y = \sqrt{\frac{1}{n}\sum(a\mu_y + b - aX_i - b)^2}$$
$$\sigma_y = \sqrt{\frac{1}{n}\sum(a\mu_x - aX_i)^2}$$
$$\sigma_y = a\sqrt{\frac{1}{n}\sum(\mu_x - X_i)^2}$$
$$\sigma_y = a\sigma_x$$

ANSWERS

8. Ans. C.

for non-trivial solution

$$|A| = 0$$

where $|A| = \begin{bmatrix} p & q & r \\ q & r & p \\ r & p & q \end{bmatrix} = p * (rq - p^2) - q * (q^2 - pr) + r * (qp - r^2)$

$= prq - p^3 - q^3 + prq + prq - r^3$

$= 3prq - p^3 - q^3 - r^3$

$= -(p + q + r)^3 + 3(p + q + r)(pq + qr + pr)$

now if you check the options the only options where each individual condition can make $|A| = 0$ zero is C.

9. Ans. C.

I is not correct

$x + y + z = 1$

$x + y + z = 0$

Has no solution, when no of equations is less than no of variables.

II is not correct

Eg:

$x - 2y = 2$

$2x + 8y = 16$

$x + y = 5$

Has a solution (x = 4, y =1)

III is correct

Eg:

$x + y = 4,$

$x + 2y = 0$

Has solutions (x=6, y = -2)

10. Ans. A.

Let's define a new function g,

g(y) = f(y) -f(y+1)

Since function f is continuous in [0,2], therefore g would be continuous in [0,1]

g(0) = -2, g(1) = 2

since g is continuous and goes from negative to positive value in [0,1]. Therefore at some point g would be 0 in (0,1).

g=0 => f(y) = f(y+1) for some y in (0,1).

Therefore, the correct answer is (A).

11. Ans. B.

Here f(x) can be either an even function or an odd function.

If f(x) was an odd function, then f(x) + f(-x) = 0, but here it has been given that it has degree 10. Then, it must be an even function.

Therefore, f(x) = f(-x)

=> f'(x) = -f'(-x)

Also, it has been mentioned that g(x) is the derivative of f(x).

So, g(x) = f'(x) and g(-x) = -f'(-x)

ANSWERS

=> g(x) – g(-x) = f'(x) – (-f'(-x))
=> g(x) – g(-x) = f'(x) + f'(x)
=> g(x) – g(-x) = 2 * f'(x)
But, f'(x) will have degree 9

12. Ans. D.

$$\int_0^{2\pi} |x\sin x|\, dx = \int_0^{\pi} x\sin x dx + \left(-\int_{\pi}^{2\pi} x\sin x dx\right)$$

$$= \int_0^{\pi} x\sin x dx - \int_{\pi}^{2\pi} x\sin x dx.$$

$$\int_0^{\pi} u dv = uv - \int_0^{\pi} v du$$

Here $u = x, du = dx, dv = \sin x dx$, so $v = -\cos x$

$$\therefore \int_0^{\pi} x\sin x dx = [-x\cos x]_0^{\pi} + \int_0^{\pi} \cos x dx$$

$$= \pi + [\sin x]_0^{\pi}$$
$$= \pi$$

Now, $\int_{\pi}^{2\pi} x\sin x = [-x\cos x]_{\pi}^{2\pi} + \int_{\pi}^{2\pi} \cos x dx$

$$= -3\pi + [\sin x]_{\pi}^{2\pi}$$
$$= -3\pi$$

So, given integral = $\pi - (-3\pi) = 4\pi$

So, k = 4.

13. Ans. B.
Derivative of $g_X(z)$ evaluated at z=1 gives expectation E(X) of X.
Therefore, take derivative of $g_Y(z)$ with respect to z, and plug in z=1
Derivative is N.β.$(1 - \beta + \beta z)^{(N-1),}$ plug in z=1, gives Nβ.

14. Ans. D.
Clearly 13 = 1 × 10 + 3 and $36 = 3\times 10 + 6 \Rightarrow base\ b = 10$
The quadratic equation with solutions x = 5 and x = 6 is $x^2 - 11x + 30 = 0$
According to the given condition, we have $b + 3 = 11$ and $3b + 6 = 30 \Rightarrow b = 8$
Answer is 8.
Alternate solution:
$x^2 - 13x + 36 = 0$ (given quadratic equation)
In base b, $13 = 1\times b^1 + 3\times b^0 = b + 3$ and $36 = 3\times b^1 + 6\times b^0 = 3b + 6$
So the equation becomes $x^2 - (b+3)x + +(3b+6) = 0$
Since x = 5 is a solution
$\therefore 5^2 - (b+3)5 + (3b+6) = 0 \Rightarrow b = 8$
Similarly, by putting x = 6, we get b = 8

ANSWERS

15. Ans. C.

Total Possibilities = $(10)^k$ because every digit has 10 options from 0 to 9.
Possibility of not containing any digit 0,5,9 = $(7)^k$, now every digit has 7 options.
Asked probability $=(7)^k/(10)^k=(0.7)^k$
So C is the answer.

16. Ans. B.

{integers between 1 to 500 divisible by 3}
{integers between 1 to 500 divisible by 5}
{integers between 1 to 500 divisible by 7}
To find number of integers between 1 to 500 that are divisible by 3 or 5 or 7 is to find $|D_3 \cup D_5 \cup D_7|$

$$= [|D_3|+|D_5|+|D_7|]-[|D_3 \cap D_5|+|D_5 \cap D_7|]+[|D_3 \cap D_5 \cap D_7|]$$

$$= \left(\left[\frac{500}{3}\right]+\left[\frac{500}{5}\right]+\left[\frac{500}{7}\right]\right)-\left(\left[\frac{500}{15}\right]+\left[\frac{500}{21}\right]+\left[\frac{500}{35}\right]+\left[\frac{500}{105}\right]\right)$$

$$= (166+100+71)-(33+23+14)+4$$

$$= 337-70+4=271$$

17. Ans. B.

$\rho(A)<n|A|=0 \Rightarrow$
one eigen value must be '0'
$\in[-5,5]$
∴ (I) is true

Let $A=\begin{bmatrix}5 & 0 & 0\\ 0 & -5 & 0\\ 0 & 0 & 0\end{bmatrix} \Rightarrow \sum_{i=1}^{3}\sum_{ij}^{3} A_{ij}^2=50$ and $\rho(A)=2$

but eigen values of A are 0, −5,5
∴ The eigen value with the largest magnitude is not greater than 5

For and Let $A=\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix} \Rightarrow$ eigen values = 5,5

∴ One eigen value must be in [−5,5] and largest eigen value magnitude is not greater than 5
∴ (II) is false

18. Ans. C.

Since the scalars are not all zero
∴ The column vectors for i =1,2...,n are linearly dependent

$\Rightarrow |A|=0$ and $b=\sum_{i=1}^{n} a_i \Rightarrow Ax=b$ has infinitely many solutions.

19. Ans. B.

$\forall x(\exists y R(x,y)) \Rightarrow \exists y \exists x R(x,y)$

$\exists y \forall x R(x,y) \Rightarrow \forall x \exists y R(x,y)$

$\forall x \exists y R(x,y) \not\Rightarrow \exists y \forall x R(x,y)$

$\neg \exists x(\forall y \neg R(x,y)) \Leftrightarrow \forall x \exists y R(x,y)$

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